JHEP HYPER VERSION

Received: *April 27, 2000*, Accepted: *May 26, 2000*

# Testing stability of M-theory on an $S^1/\mathbb{Z}_2$ orbifold

**Axel Krause**

*Humboldt-Universität, Institut für Physik*
*D-10115 Berlin, Germany*
*E-mail:* axkrause@qft18.physik.hu-berlin.de

Abstract: We analyse perturbatively, whether a flat background with vanishing $G$-flux in Hořava-Witten supergravity represents a vacuum state, which is stable with respect to interactions between the ten-dimensional boundaries, mediated through the $D = 11$ supergravity bulk fields. For this, we consider fluctuations in the graviton, gravitino and 3 form around the flat background, which couple to the boundary $E_8$ gauge-supermultiplet. They give rise to exchange amplitudes or forces between both boundary fixed-planes. In leading order of the $D = 11$ gravitational coupling constant $\kappa$, we find an expected trivial vanishing of all three amplitudes and thereby stability of the flat vacuum in the static limit, in which the centre-of-mass energy $\sqrt{s}$ of the gauge-multiplet fields is zero. For $\sqrt{s} > 0$, however, which could be regarded a vacuum state with excitations on the boundary, the amplitudes neither vanish nor cancel each other, thus leading to an attractive force between the fixed-planes in the flat vacuum. A ground state showing stability with regard to boundary excitations, is therefore expected to exhibit a non-trivial metric. Ten-dimensional Lorentz-invariance requires a warped geometry. Finally, we extrapolate the amplitudes to the case of coinciding boundaries and compare them to the ones resulting from the weakly coupled $E_8 \times E_8$ heterotic string theory at low energies.

Keywords: Field Theories in Higher Dimensions, M-Theory, Supergravity Models.

JHEP05(2000)046

## Contents



## 1. Introduction and motivation

With the discovery of M-theory on an $S^1/\mathbb{Z}_2$-orbifold [1] and its concrete low-energy realization as Hořava-Witten supergravity [2], i.e. $D = 11$ supergravity coupled to two super Yang-Mills theories with $E_8$ gauge group, living on two separate boundaries of space-time, the vexing problem of predicting the correct magnitude for the $D = 4$ Newton-constant $G_N$ could be addressed anew. While the heterotic string theory predicts a value for $G_N$ which is generically too large by a factor of 400, M-theory on $S^1/\mathbb{Z}_2$ could account for the correct value by adjusting an additional parameter, the distance $d$ between the two boundaries, roughly at the inverse of the GUT-scale

$10^{16}$ GeV [3]. Similar to the weak-strong coupling relationship between the type-IIA-string and M-Theory, it was found for M-theory on $S^1/\mathbb{Z}_2$, that $g_s^{2/3} = d/\pi$, where $g_s$ is the heterotic string coupling constant. Since in the limit $d \to 0$ of coinciding boundaries the string coupling turns out to be weak, it is believed that we should recover the usual heterotic string theory with gauge group $E_8 \times E_8$. Thus M-theory on $S^1/\mathbb{Z}_2$ has also been termed heterotic M-theory. Since even more phenomenological virtues of heterotic M-theory were discovered, e.g. it avoids the problem of small gaugino masses [4], it is an interesting question to ask for its stability — in particular if the boundary fields have non-vanishing energy.

Since the boundaries of the theory maintain an $E_8$ super Yang-Mills gauge theory, respectively, a non-vanishing energy-momentum tensor gets induced on each of them. Because gravity couples to any energy-momentum tensor, an interaction between the boundaries mediated by gravitons in the bulk is inevitable. This interaction should be attractive, as can be expected from classical gravity. But furthermore the $D = 11$ supergravity bulk theory allows for gravitino and 3-form exchanges, which do couple to the boundary fields as well, due to the underlying supersymmetry. Therefore we have to analyze for heterotic M-theory on the proposed [1, 2] $\mathbb{R}^{1,9} \times S^1/\mathbb{Z}_2$ space-time, whether all these contributions cancel each other, leading to a stable configuration or not. Since it is not known how to quantize $D = 11$ supergravity consistently, we will restrict ourselves to a purely classical analysis of the stability problem. Remembering the well-known derivation of the complete Coulomb or Newton potential from tree-level photon or graviton exchange diagrams, this should not amount to a severe restriction. Noting that the construction of Hořava-Witten supergravity has been achieved as an expansion in powers of small $\kappa^{2/3}$, where $\kappa$ is the $D = 11$ gravitational coupling constant, we are furthermore allowed to examine the interactions between the two boundaries in linearized gravity to leading order in $\kappa$ and discard higher-order contributions as subleading corrections.

One may be inclined to argue that the situation should be similar to the analogous case of an interaction between two D-branes of type-II string theory (see [5] for a review). There the repulsion of the RR-field compensates exactly the attraction originating from graviton and dilaton exchange. However, in order to reach that conclusion we have to avail ourselves of the duality between the closed-string tree-level cylinder amplitude and the open-string 1-loop annulus diagram. Only through the latter is it possible to see the cancellation by appealing to Jacobi's *aequatio identica satis abstrusa*. This is in accord with the common lore that supersymmetry leads to cancellations between fermionic and bosonic loop-contributions (most prominently applied to the solution of the weak hierarchy problem). In contrast to the type-II case, heterotic M-theory has been formulated only as a classical field theory, so far. Therefore, we have to deal with genuine tree diagrams (without any duality to some possibly vanishing loop counterpart), for which, even in a supersymmetric theory, there is *a priori* no reason that they add up to zero. If one could consistently (note



JHEP05(2000)046

that supergravity is non renormalizable) work out the Casimir-energy at the one-loop level, then first this could be expected to vanish on account of the presence of supersymmetry in the bulk. Second this would constitute only a small quantum correction of order $\hbar$ compared to the leading order tree-level result obtained below. For these reasons we will not explore the quantum Casimir-energy further in this paper.

It is interesting to consult the supersymmetry variations for the bulk fields. In heterotic M-theory, the incorporation of $E_8$ super Yang-Mills theories on the two orbifold fixed-planes, simultaneously requires the augmentation of the susy-variations of the bulk-fields [2]. The additional contributions have support on the fixed-planes only and are solely built out of the boundary-fields. For the particular flat Minkowski background with vanishing G-flux, which we will examine later on, the bulk contributions completely vanish, since flat space does not break any supersymmetry at all. The only non-vanishing contributions for constant Majorana-spinor $\eta$ derive from the boundary fields

$$\delta C_{11BC} = -\frac{1}{24\sqrt{2}\,\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\delta\left(x_i^{11}-d_i\right)\bar{\eta}\,A^a_{[B}\Gamma_{C]}\,\chi^a_i\,, \tag{1.1}$$

$$\delta\Psi_A = -\frac{1}{576\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\delta\left(x_i^{11}-d_i\right)\left(\bar{\chi}^a_i\,\Gamma_{BCD}\chi^a_i\right)\left(\Gamma_A{}^{BCD}-6\,\delta^B_A\,\Gamma^{CD}\right)\eta\,, \tag{1.2}$$

$$\delta\Psi_{11} = \frac{1}{576\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\delta\left(x_i^{11}-d_i\right)\left(\bar{\chi}^a_i\,\Gamma^{ABC}\chi^a_i\right)\Gamma_{ABC}\,\eta\,. \tag{1.3}$$

In momentum-space these contributions will vanish in the case of equal momenta of the boundary fields. Contracting $\delta C_{11BC}$ with the momentum $p_2^C$ of the gauge-field $A^a_B$, we get an expression proportional to

$$\bar{\eta}\Big(A^a_B(p_2)\;\not{p}_2 - p_2\cdot A^a(p_2)\Gamma_B\Big)\chi^a(p_3)\,. \tag{1.4}$$

Choosing the Lorentz-gauge, the second term disappears, whereas the first term gives zero, when we choose $p_2 = p_3 = p$ on account of the massless Dirac-equation $\not{p}\chi^a(p) = 0$. For the last two gravitino-variations, we note that

$$\bar{\chi}^a(p)\,\Gamma^{ABC}\chi^a(p') = -\bar{\chi}^a(p')\,\Gamma^{ABC}\chi^a(p)\,, \tag{1.5}$$

from which we easily recognize, that the gaugino bilinear $\bar{\chi}^a\Gamma^{ABC}\chi^a$ also vanishes in the limit of coinciding momenta for $\bar{\chi}^a$ and $\chi^a$. In kinematical language, coinciding momenta mean a vanishing center-of-mass energy squared $s = 0$. Hence, in this limit we expect to find no interaction between the boundaries, for the assumed flat vacuum.

The interaction amplitudes will depend on the parameter $d$, representing the distance between the two boundaries in the eleventh direction. In case that we can still trust Hořava-Witten supergravity not only for large values of $d$ but also for small values, then according to the conjecture, the $d \to 0$ limit of the above amplitudes should correspond to the low-energy limit of heterotic string amplitudes.

Consequently we will derive the adequate string expressions describing an exchange of $D = 10$ supergravity multiplet fields in the low-energy limit $\alpha' s, \alpha' t, \alpha' u \ll 1$ and compare them with our M-theory amplitudes evaluated at $d = 0$. Naively, one would not expect complete agreement of the two sets of amplitudes, since a large $d$ compared to the eleven-dimensional Planck-scale is a necessary condition for the validity of the effective Hořava-Witten supergravity.

The rest of the paper is organized as follows. In section 2 we will apply the background field method to Hořava-Witten supergravity with $\mathbb{R}^{1,9} \times S^1$ chosen as the background.[1] After gauge fixing the relevant symmetries, we obtain the bulk propagators and determine the leading order couplings of the boundary fields to the bulk supergravity fields. section 3 continues with the calculation of the amplitudes for graviton, gravitino and 3-form exchange between the boundaries and a subsequent analysis of the stability of the theory. In section 4 we derive the $d \to 0$ limits of the amplitudes and compare them with their analogues from low-energy heterotic string theory. Section 5 finally ends with a summary and a conclusion.

## 2. Expansion of Hořava-Witten supergravity around $\mathbb{R}^{1,9} \times S^1$ background

As advocated in [1, 2] we should choose $\mathbb{R}^{1,9} \times S^1$ as our $D = 11$ space-time manifold, where the eleventh coordinate $x^{11}$ is curled up to a circle which we parameterize by $[-d, d]$ with $d \sim -d$ to be identified. Furthermore we have to impose the constraint, that the fields be invariant under the reflection $x^{11} \to -x^{11}$. This so-called "upstairs" formulation, which we shall employ here, has the advantage that one can work with a smooth manifold, whereas in the equivalent alternative "downstairs" formulation one would have to deal with a bounded manifold $\mathbb{R}^{1,9} \times S^1/\mathbb{Z}_2 = \mathbb{R}^{1,9} \times [0, d]$ and prescribe suitable boundary conditions. In the latter approach the boundary is given by the two codimension one fixed planes of the reflection map, situated at $x^{11} = 0$ and $d$.

The construction of Hořava-Witten supergravity proceeds by a power series in the expansion parameter $\kappa^{2/3}$. To lowest order, one starts with the action of $N = 1$, $D = 11$ supergravity [6]

$$
\begin{aligned}
S_{\text{bulk}} = \int_{\mathbb{R}^{1,9}\times S^1} d^{11}x \, \frac{\sqrt{-g}}{\kappa^2} \Bigg[ &- \frac{R}{2} - \frac{1}{2} \bar{\Psi}_I \Gamma^{IJK} D_J \Psi_K - \frac{1}{2 \times 4!} G_{IJKL} G^{IJKL} - \\
&- \frac{\sqrt{2}}{192} \left( \bar{\Psi}_I \Gamma^{IJKLMN} \Psi_N + 12 \, \bar{\Psi}^J \Gamma^{KL} \Psi^M \right) G_{JKLM} - \\
&- \frac{\sqrt{2}}{3456} \epsilon^{I_1 \dots I_{11}} C_{I_1 I_2 I_3} G_{I_4 \dots I_7} G_{I_8 \dots I_{11}} \Bigg] + \mathcal{O}\left(\Psi^4\right) \qquad (2.1)
\end{aligned}
$$

[1] We are working in the "upstairs" formulation [2], regarding the theory as an orbifold on $\mathbb{R}^{1,9} \times S^1$ with a $\mathbb{Z}_2$ symmetry imposed on the fields.

for the bulk multiplet, consisting of elfbein $e^{\bar{I}}{}_I$, gravitino $\Psi_I$ and 3-form $C_{IJK}$. We use $I,\ldots,N/A,\ldots,F$ to represent $D = 11/D = 10$ space-time indices and $\bar{I},\ldots,\bar{N}/\bar{A},\ldots,\bar{F}$ for their tangent space analogues. Moreover we define $\bar{\Psi}_\alpha = C_{\alpha\beta}\Psi^\beta$, where the real, anti-symmetric charge conjugation matrix $C_{\alpha\beta}$ obeys $C^{\alpha\beta}C_{\beta\gamma} = \delta^\alpha_\gamma$ (see the appendix for our conventions). The covariant derivative of the gravitino, the spin connection $\Omega_{J\bar{L}\bar{M}}$ and the 4-form field strength $G_{IJKL}$ are defined as

$$\begin{aligned}
D_J\Psi_K &= \partial_J\Psi_K + \frac{1}{4}\,\Omega_{J\bar{L}\bar{M}}\,\Gamma^{\bar{L}\bar{M}}\Psi_K\,,\\
\Omega_{J\bar{L}\bar{M}} &= \frac{1}{2}\left(e_{\bar{L}}{}^L\tilde{\Omega}_{JL\bar{M}} - e_{\bar{M}}{}^L\,\tilde{\Omega}_{JL\bar{L}} - e_{\bar{L}}{}^L\,e_{\bar{M}}{}^M\,e^{\bar{J}}{}_J\,\tilde{\Omega}_{LM\bar{J}}\right),\\
\tilde{\Omega}_{JL\bar{M}} &= \partial_J e_{\bar{M}L} - \partial_L e_{\bar{M}J}\,,\\
G_{IJKL} &= 4!\,\partial_{[I}\,C_{JKL]}\,.
\end{aligned}$$

Gravitational anomalies arise on the $D = 10$ fixed planes. The factorizable $(\mathrm{tr}\,R^2)^3$, $\mathrm{tr}\,R^2\,\mathrm{tr}\,R^4$ terms can be cured by a Green-Schwarz-like mechanism whereas the irreducible $\mathrm{tr}\,R^6$ part of the anomaly necessitates the introduction of 496 $D = 10$ vector-supermultiplets for compensation. Due to the $\mathbb{Z}_2$-symmetry they have to be compartmentalized equally to 248 multiplets on each fixed plane, which singles out the $E_8$ gauge group for the super Yang-Mills theories on each boundary. Altogether the $D = 10$ boundary action for the $E_8$ vector supermultiplet,[2] comprising the gauge field $A^a$ and the gaugino $\chi^a$, coupled to the bulk supergravity in a locally supersymmetric fashion, reads [2] $(i = 1, 2)$

$$\begin{aligned}
&S_{i,\text{bound}}\left(x^{11} = d_i\right) =\\
&= \int_{\mathbb{R}^{1,9}} d^{10}x_i\,\frac{1}{(4\pi)^{5/3}\kappa^{4/3}}\,\sqrt{-g}\,\times\\
&\quad\times\left[-\frac{1}{4}\,F^a_{iAB}\,F^{aAB}_i - \frac{1}{2}\,\bar{\chi}^a_i\,\Gamma^A D_A\,\chi^a_i - \frac{1}{4}\,\bar{\Psi}_A\,\Gamma^{BC}\,\Gamma^A F^a_{iBC}\,\chi^a_i +\right.\\
&\qquad + \bar{\chi}^a_i\,\Gamma^{ABC}\chi^a_i\left[\frac{\sqrt{2}}{48}\,G_{ABC11} + \frac{1}{32}\,\bar{\Psi}_A\Gamma_{BC}\Psi_{11} + \frac{1}{32}\bar{\Psi}^D\Gamma_{DABC}\Psi_{11} +\right.\\
&\qquad\qquad + \frac{1}{128}\left(3\bar{\Psi}_A\Gamma_B\Psi_C - \bar{\Psi}_A\Gamma_{BCD}\Psi^D - \frac{1}{2}\,\bar{\Psi}_D\Gamma_{ABC}\Psi^D -\right.\\
&\qquad\qquad\qquad \left.\left.\left. - \frac{13}{6}\bar{\Psi}^D\Gamma_{DABCE}\Psi^E\right)\right]\right],
\end{aligned} \tag{2.2}$$

where $d_1 = 0, d_2 = d$ describe the two-fixed plane positions. The non-abelian field strength $F^a_{iAB}$ and the covariant derivative for the gaugino are defined as usual as

$$\begin{aligned}
F^a_{iAB} &= \partial_A A^a_{iB} - \partial_B A^a_{iA} + f^a{}_{bc}\,A^b_{iA}\,A^c_{iB}\,,\\
D_A\chi^a_i &= \partial_A\chi^a_i + f^a{}_{bc}\,A^b_{iA}\chi^c_i + \frac{1}{4}\,\Omega_{A\bar{B}\bar{C}}\,\Gamma^{\bar{B}\bar{C}}\chi^a_i\,.
\end{aligned}$$

[2]For $E_8$ the Lie-Algebra index $a$ runs from 1 to 248.

Actually, on the boundary the 4-form field strength receives an additional contribution resulting in $G_{ABC11} = 4!\partial_{[A}c_{BC11]} - \frac{\kappa^{2/3}}{\sqrt{2}(4\pi)^{5/3}}\delta(x^{11}-d_i)\,\omega_{ABC}(A^a_A)$, where $\omega_{ABC}$ is a Chern-Simons term depending solely on the $E_8$ gauge field $A^a_A$. But since the second term does not yield a coupling to the bulk fields, it is irrelevant for our purposes and will be neglected in the following. Furthermore the gauginos possess positive chirality $\Gamma^{10}\chi^a = \chi^a$. The gauge coupling constant $\lambda$ has been eliminated from (2.2) by means of the relation[3] $\lambda^2 = 4\pi(4\pi\kappa^2)^{2/3}$. It results from demanding that on the boundary the non-vanishing gauge variation of the Chern-Simons term $C \wedge G \wedge G$ should be cancelled by the gauge anomaly for the $D = 10$ Majorana-Weyl gaugino $\chi^a$. Though gauge invariance cannot be established at the classical level, it should be valid at the quantum level, at least. The fixed plane gauge action (2.2) is the second-order term in the power series expansion in $\kappa^{2/3}$. Unfortunately, in the next higher-order infinities arise in the form of $\delta(0)$ terms occuring in the lagrangian. Formally, these infinities cancel in verifying supersymmetry. Nevertheless, to arrive at reliable results, one is forced to truncate the action at this order consistently. This will become important below.

From the field-theoretic point of view, we have to look for small fluctuations of the bulk fields in order to mediate interactions between the boundary fields. This will be achieved by using the background field method [8], according to which we split the bulk fields $e^{\bar{M}}{}_M, \Psi_M, C_{MNP}$ into a fixed classical background $\tilde{e}^{\bar{M}}{}_M, \tilde{\psi}_M, \tilde{c}_{MNP}$ and the quantum fields $f^{\bar{M}}{}_M, \psi_M, c_{MNP}$, which propagate on this background

$$
\begin{aligned}
e^{\bar{M}}{}_M &= \tilde{e}^{\bar{M}}{}_M + \kappa f^{\bar{M}}{}_M \longrightarrow g_{MN} = \tilde{g}_{MN} + \kappa h_{MN}\,,\\
\Psi_M &= \tilde{\psi}_M + \kappa\psi_M\,,\\
C_{MNP} &= \tilde{c}_{MNP} + \kappa c_{MNP}\,.
\end{aligned}
$$

The action is then expanded around the background fields into a power series of the quantum fields. The further multiplication with the $D = 11$ gravitational coupling constant $\kappa$ has been chosen to give the fluctuations ordinary kinetic terms in the action. Hence the expansion in quantum fields is also one in powers of $\kappa$. In the following every index will be raised or lowered by means of the background elfbein or metric.

With the chosen parameterization of the circle of radius $R = d/\pi$, our background $\mathbb{R}^{1,9} \times S^1$ is described locally by a flat elfbein[4]

$$\tilde{e}^{\bar{M}}{}_M = \delta^{\bar{M}}_M\,.$$

[3]Originally, the relation appeared as $\lambda^2 = 2\pi(4\pi\kappa^2)^{2/3}$ in [2]. We employ a further factor of 2, which was found in [7]. Since $\lambda$ will enter our analysis only through an overall factor, which is the same for all amplitudes, this ambiguity will not have an influence on our conclusions.

[4]For flat space the curved space-time indices $M, N, \ldots$ and the tangent space indices $\bar{M}, \bar{N}, \ldots$ coincide and need not to be distinguished subsequently.

In order not to break spontaneously the $D = 10$ Lorentz symmetry, the background gravitino field $\tilde{\psi}_M$ as well as the background 3-form $\tilde{c}_{MNP}$ must vanish

$$\tilde{\psi}_M = \tilde{c}_{MNP} = 0 \,.$$

As we will point out in the following, it is advantageous (though not necessary) if the background fields fulfill the equations of motion as is the case for our flat background.

The expansion of the bulk action then proceeds as

$$\int d^{11}x \Bigg( \frac{1}{\kappa^2} \,[\text{classical Sugra action}] + \frac{1}{\kappa} \,[\text{linear in quantum fields } h, \psi, c] + \\ + [\text{quadratic terms for } h, \psi, c] + \mathcal{O}(\kappa) \Bigg) \,.$$

For our zero-curvature background the leading term vanishes. Since the coefficients of $h, \psi, c$ in the $1/\kappa$-term are precisely the variational derivatives of the action with respect to the classical fields, they will vanish when the background satisfies the equations of motion — as in our case. In order to extract the propagators from the quadratic part, we have, according to the usual Faddeev-Popov procedure, to fix all the gauge symmetries of the quantum fields and introduce corresponding ghosts. However, since our analysis is intended to be classical, i.e. at tree-level, we can neglect the ghost fields. $N = 1$, $D = 11$ supergravity possesses four-different gauge symmetries, which are fixed as follows

- $D = 11$ general coordinate invariance $\rightarrow$ de Donder (harmonic) gauge[5]

$$\Delta\mathcal{L}_{GC} = -\frac{\tilde{e}}{2} \left( h_M{}^N{}_{;N} - \frac{1}{2} h^N{}_{N;M} \right)^2 \,.$$

- $\mathrm{SO}(1, 10)$ local Lorentz invariance $\rightarrow$ symmetric gauge

$$\Delta\mathcal{L}_{LL} = -\frac{\tilde{e}}{2\kappa^{4/(D-2)}} \left( f_{MN} - f_{NM} \right)^2 \longrightarrow -\frac{\tilde{e}}{2\kappa^{4/9}} \left( f_{MN} - f_{NM} \right)^2 \,.$$

- local abelian gauge transformations of $C_{MNP}$ $\rightarrow$ Lorentz-like gauge

$$\Delta\mathcal{L}_{Ab} = -\frac{3}{2} \left( 3! \partial^M c_{MNP} \right)^2 = -\frac{3}{2} (3!)^2 \, \partial_I \, c^{IJK} \partial^L c_{LJK} \,.$$

- local $N = 1$ supersymmetry $\rightarrow$ $\Gamma^M \psi_M = 0$ fixing[6] [9]

$$\Delta\mathcal{L}_{SS} = -\frac{\zeta}{2} \, \bar{\psi}_M \Gamma^M \Gamma^N \partial_N \Gamma^L \psi_L \,.$$

[5] As usual we define $\tilde{e} = \sqrt{-\tilde{g}}$.

[6] $\zeta$ is a free gauge parameter which we will set later on equal to $-9/4$ for simplicity.

Note that $\Delta\mathcal{L}_{LL}$ is a purely algebraic term and thus does not contribute to the propagator. From $e^{\bar{M}}{}_M e^{\bar{N}}{}_N \eta_{\bar{M}\bar{N}} = g_{MN}$, $\tilde{e}^{\bar{M}}{}_M \tilde{e}^{\bar{N}}{}_N \eta_{\bar{M}\bar{N}} = \tilde{g}_{MN}$ and $e^{\bar{M}}{}_M = \tilde{e}^{\bar{M}}{}_M + \kappa f^{\bar{M}}{}_M$ plus the above gauge fixing of the Lorentz-symmetry, which gauges the anti-symmetric part of $f_{MN}$ away, we conclude that $h_{MN} = 2f_{MN} + \mathcal{O}(\kappa)$. Therefore we can express the elfbein fluctuations in terms of the metric fluctuations. The quadratic terms of the bulk action then lead to the following propagators in momentum space, valid for flat, unbounded $D = 11$ Minkowski-space (the compactification of the eleventh coordinate on the circle will manifest itself, later on, in a replacement of $p^{11}$ by discrete values $p^{11}_m; m \in \mathbb{Z}$):
Graviton $h_{MN}$:

$$\Delta_{M_1M_2,N_1N_2}(P) = -2\left(\eta_{M_1N_1}\eta_{M_2N_2} + \eta_{M_1N_2}\eta_{M_2N_1} - \frac{2}{9}\,\eta_{M_1M_2}\eta_{N_1N_2}\right)\frac{1}{P^2}\,, \qquad (2.3)$$

Gravitino[7] $\psi_M$:

$$\begin{aligned} \Delta_{MN}{}^{\alpha\beta}(P) &\equiv i\left(\tilde{\Delta}_{MN}\right)^{\alpha}{}_{\gamma}(P)\left(-C^{\gamma\beta}\right) & (2.4)\\ &= \frac{i}{9}\left[7\eta_{MN}\not{P} - \Gamma_N\not{P}\,\Gamma_M + \left(4+\frac{9}{\zeta}\right)\frac{P_M\not{P}P_N}{P^2}\right]^{\alpha}{}_{\gamma}\times & (2.5)\\ &\quad\times\left(-C^{\gamma\beta}\right)\frac{1}{P^2} \overset{\zeta=-9/4}{\longrightarrow} \frac{i}{9}\left[7\eta_{MN}\not{P} - \Gamma_N\not{P}\,\Gamma_M\right]^{\alpha}{}_{\gamma}\left(-C^{\gamma\beta}\right)\frac{1}{P^2}\,, \end{aligned}$$

3-form $c_{MNP}$:

$$\Delta_{M_1M_2M_3,N_1N_2N_3}(P) = \frac{1}{(3!)^2}\frac{\eta_{M_1[N_1}\eta_{|M_2|N_2}\eta_{|M_3|N_3]}}{P^2}\,, \qquad (2.6)$$

where $P = (p^A, p^{11})$ denotes the eleven-dimensional momentum.

The expansion of the boundary action reads schematically

$$\int d^{10}x_i\Big(\frac{1}{\kappa^{4/3}}\,[\text{pure SYM}] + \\ + \frac{1}{\kappa^{1/3}}\,[\text{bulk-boundary interaction terms, linear in } h,\psi,c] + \mathcal{O}\left(\kappa^{2/3}\right)\Big)\,.$$

Since the leading $1/\kappa^{4/3}$ contribution does not contain any bulk quantum field, it cannot contribute to the boundary-boundary interaction and is therefore of no interest to us. The $1/\kappa^{1/3}$ terms comprise the relevant interaction terms, whereas higher-order $\kappa^{2/3}$ expressions have to be skipped for two reasons. First, $\kappa^{2/3}$ terms would introduce couplings quadratic in $h,\psi,c$. Either these would finally lead to loop diagrams of order $\kappa^{4/3}$, which have to be neglected, since we restrict ourselves to a tree-level analysis. Or in combination with the couplings linear in $h,\psi,c$, they would give rise to order 1 tree diagrams. These are suppressed by a factor of $\kappa^{2/3}$ against

[7] $\alpha,\beta,\ldots$ denote $\mathrm{SO}(1,10)$ spinor indices.

the leading diagrams of order $1/\kappa^{2/3}$ and therefore can be neglected, too. Second, the boundary action (2.2) has been constructed only up to order $1/\kappa^{4/3}$. The next higher-order in the power series expansion in $\kappa^{2/3}$ would involve $1/\kappa^{2/3}$ terms. However, it has been argued in [2], that at this order expressions containing $\delta(0)$ show up. Therefore, in a consistent truncation of the theory, we have to skip these higher contributions altogether. In the expansion around the classical background, the bulk field fluctuations $h, \psi, c$ come equipped with an additional power of $\kappa$. Summarizing, a consistent truncation implies throwing away all bulk-boundary interactions of order $\kappa^{1/3}$ or higher. In particular, the above $\kappa^{2/3}$ contributions have to be omitted. Thus, the remaining $1/\kappa^{1/3}$ interaction terms are given by[8]

$$\begin{aligned}
S^{(1)}_{i,\text{bound}}\left(x^{11}=d_i\right) &= \\
&= \frac{1}{(4\pi)^{5/3}\kappa^{1/3}} \times \int d^{10}x_i \Big[ -\frac{1}{8} F_i^{aCD} F^a_{iCD}\, h^A{}_A + \frac{1}{2} F^a_{iAC}\, F^{a\,C}_{iB} h^{AB} + \\
&\qquad + \frac{1}{8}\bar{\chi}^a_i\,\Gamma^A\,\Gamma^{BC}\chi^a_i\,\partial_{[A}h_{B]C} - \frac{1}{16}\bar{\chi}^a_i\,\Gamma^A\,\Gamma^{BC}\chi^a_i\,\partial_{[B}h_{C]A} - \\
&\qquad - \frac{1}{4}\bar{\psi}_A\,\Gamma^{BC}\Gamma^A\chi^a_i F^a_{iBC} + \frac{1}{\sqrt{2}}\bar{\chi}^a_i\,\Gamma^{ABC}\chi^a_i\,\partial_{[A}c_{BC11]}\Big] .
\end{aligned}$$

It can be read off that we obtain a 5-point vertex $AAAAh$, two 4-point vertices $AA\chi\psi$, $AAAh$ and four 3-point vertices $\chi\chi c$, $A\chi\psi$, $\chi\chi h$, $AAh$. The 5 vertex has a group-theoretic factor which consists of a sum of terms like $\sum_{e=1}^{248}(f_{eac}\,f_{ebd}+f_{ead}\,f_{ebc})$, where $f_{abc}$ are the $E_8$ structure constants. The 4 vertices are simply proportional to $f_{abc}$. Since finally in our amplitudes we will sum over all group indices of the external boundary fields (we sum over all possible exchange amplitudes between the two boundaries), the 5- and 4-vertices give no contribution due to the antisymmetry of the structure constants. If, therefore, we concentrate merely on the 3-vertices, the relevant couplings are

$$\begin{aligned}
S^{(1)}_{i,\text{bound}}\left(x^{11}=d_i\right) &= \frac{1}{(4\pi)^{5/3}\kappa^{1/3}} \times \\
&\times \int d^{10}x_i \Big[\frac{1}{2}\Big( -\partial^C A_i^{a,D}\partial_{[C}A^a_{iD]}\eta_{AB} + \partial_A A^a_{iC}\partial_B A_i^{a,C} + \\
&\qquad + \partial_C A^a_{iA}\,\partial^C A^a_{iB} - 2\,\partial_C A^a_{iA}\partial_B\, A_i^{a,C}\Big) h^{AB} + \\
&\qquad + \frac{1}{8}h_{AC}\partial_B\left(\bar{\chi}^a_i\Gamma^A\Gamma^{BC}\chi^a_i\right) - \frac{1}{2}\bar{\psi}_A\Gamma^{BC}\Gamma^A\chi^a_i\partial_B A^a_{iC} + \\
&\qquad + \frac{1}{\sqrt{2}}\bar{\chi}^a_i\Gamma^{ABC}\chi^a_i\partial_{[A}c_{BC11]}\Big] .
\end{aligned}$$

Since every term comprises exactly two boundary fields, it is convenient for the later comparison with the string amplitudes to rescale the super Yang-Mills fields $A^a_A, \chi^a$

[8]By $x_1, x_2$ we denote the $D = 10$ flat coordinates of the two boundaries, respectively.

which bear mass dimensions $[A] = 1, [\chi] = 3/2$ to

$$B^a_A := \frac{1}{(4\pi)^{5/6}\kappa^{2/3}}\, A^a_A\,, \qquad \lambda^a := \frac{1}{(4\pi)^{5/6}\kappa^{2/3}}\, \chi^a\,.$$

The fields $B^a_A$ and $\lambda^a$ have $D$-dimensional mass dimensions $[B^a_A] = (D-2)/2$ and $[\lambda^a] = (D-1)/2$, i.e. 4 and 9/2 for $D = 10$. This rescaling gives a "canonical" factor of $\kappa$ for the interaction terms, which eventually read

$$S^{(1)}_{i,\text{bound}}\left(x^{11} = d_i\right) = \kappa \int_{\mathbb{R}^{1,9}} d^{10}x_i \Big(\mathcal{L}_{i,BBh} + \mathcal{L}_{i,\lambda\lambda h} + \mathcal{L}_{i,B\lambda\psi} + \mathcal{L}_{i,\lambda\lambda c}\Big)\,, \tag{2.7}$$

where

$$\mathcal{L}_{i,BBh} = \partial_A B^a_{iB}(x_i)\, \partial_C B^a_{iD}(x_i) h_{EF}(x_i, d_i) F^{ABCDEF}\,, \tag{2.8}$$
$$\mathcal{L}_{i,\lambda\lambda h} = -\frac{1}{8}\, h_{AC}(x_i, d_i)\, \partial_B \Big[\lambda^{a\alpha}_i(x_i) C_{\alpha\beta} \left(\Gamma^C \eta^{AB} - \Gamma^B \eta^{AC}\right)^\beta{}_\gamma\, \lambda^{a\gamma}_i(x_i)\Big]\,, \tag{2.9}$$
$$\mathcal{L}_{i,B\lambda\psi} = \frac{1}{2}\, \psi^\alpha_A(x_i, d_i) C_{\alpha\beta} \left(\Gamma^{BC}\Gamma^A\right)^\beta{}_\gamma\, \lambda^{a\gamma}_i(x_i)\, \partial_B B^a_{iC}(x_i)\,, \tag{2.10}$$
$$\mathcal{L}_{i,\lambda\lambda c} = -\frac{1}{\sqrt{2}}\, \lambda^{a\alpha}_i(x_i) C_{\alpha\beta} \left(\Gamma^{ABC}\right)^\beta{}_\gamma\, \lambda^{a\gamma}_i(x_i)\, \partial_{[A} c_{BC11]}(x_i, d_i) \tag{2.11}$$

and

$$F^{ABCDEF} = \frac{1}{2}\, \eta^{A[D}\eta^{C]B}\eta^{EF} + \eta^{A[C}\eta^{D]F}\eta^{EB} + \eta^{A[E}\eta^{D]B}\eta^{CF}\,.$$

## 3. Derivation of the boundary-boundary interaction amplitudes

In order to incorporate the $\mathbb{Z}_2$ fixed point constraints and to circumvent ambiguities arising from Feynman diagrams involving Majorana fermions (which allow for twice as many Wick-contractions as Dirac fermions do), we choose not to work with Feynman rules in momentum space directly but to start with a space-time formulation of the S-matrix on $\mathbb{R}^{1,9} \times S^1$. For a tree-level boundary-boundary interaction the S-matrix reads

$$\begin{aligned} S &= -\frac{1}{2}\kappa^2 \int_{\mathbb{R}^{1,9}} d^{10}x_1 \oint dx^{11}_1 \int_{\mathbb{R}^{1,9}} d^{10}x_2 \oint dx^{11}_2\, \times \\ &\quad \times \langle f|\, T\Big(\, : \mathcal{L}_{1,\text{bound}}\left(x_1, x^{11}_1\right) \delta\left(x^{11}_1\right) :: \mathcal{L}_{2,\text{bound}}\left(x_2, x^{11}_2\right) \delta\left(x^{11}_2 - d\right) :\Big)\, |i\rangle \\ &= -\frac{1}{2}\kappa^2 \int d^{10}x_1 \int d^{10}x_2 \langle f|\, T\Big(\, : \mathcal{L}_{1,\text{bound}}(x_1, 0) :: \mathcal{L}_{2,\text{bound}}(x_2, d) :\Big)\, |i\rangle\,, \end{aligned} \tag{3.1}$$

where $\mathcal{L}_{i,bound}$ represents one of the four couplings $\mathcal{L}_{i,BBh}, \mathcal{L}_{i,\lambda\lambda h}, \mathcal{L}_{i,B\lambda\psi}, \mathcal{L}_{i,\lambda\lambda c}$ given in (2.8)–(2.11). From the eleven-dimensional perspective the fixed point constraints enter via delta-function sources which generate flat $p^{11}$-spectra in momentum space.



JHEP05(2000)046

Momentum is conserved only along the ten flat directions parallel to the boundaries, whereas there is no such conservation in the eleventh compactified direction transverse to the boundaries. This fact is also well-known from studies of radiation off D-branes [11, 10]. Therefore the kinematic variables $s, t, u$ of the scattering process are defined through the ten-dimensional momenta $p_1, p_2$ of the incoming states and $p_3, p_4$ of the outgoing states as follows

$$s = -(p_1 + p_2)^2 , \qquad t = -(p_1 - p_3)^2 , \qquad u = -(p_1 - p_4)^2 .$$

As usual $D = 10$ energy-momentum conservation implies for massless states $s+t+u = 0$. The four 3-vertices (2.8)–(2.11) can be combined into five different tree-diagrams which we will now consider in detail.

### 3.1 Graviton exchange

The first diagram is depicted in figure 1 and describes the pure graviton exchange between the boundary gauge fields. Upon substituting (2.8) into (3.1) it yields the following S-matrix contribution:

$$\begin{aligned} S_h = -\frac{1}{2}\kappa^2 \sum_{a,b,c,d=1}^{248} \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4=1}^{8} & \int d^{10}x_1 \oint dx_1^{11} \int d^{10}x_2 \oint dx_2^{11} \times \\ \times \langle 0 | \, b^d_{1\lambda_4}(p_4)\, b^c_{1\lambda_3}(p_3)\, T\Big( & : \partial_{A_1} B^{a_1}_{1B_1}(x_1)\, \partial_{C_1} B^{a_1}_{1D_1}(x_1)\, h_{E_1F_1}(x_1, 0)\, \delta\left(x_1^{11}\right) : \\ & : \partial_{A_2} B^{a_2}_{2B_2}(x_2)\, \partial_{C_2} B^{a_2}_{2D_2}(x_2)\, h_{E_2F_2}(x_2, d) \times \\ & \times \delta\left(x_2^{11} - d\right) : \Big) b^{b,\dagger}_{2\lambda_2}(p_2)\, b^{a,\dagger}_{2\lambda_1}(p_1) \, | 0 \rangle \times \\ \times F^{A_1B_1C_1D_1E_1F_1} F^{A_2B_2C_2D_2E_2F_2} . \end{aligned}$$

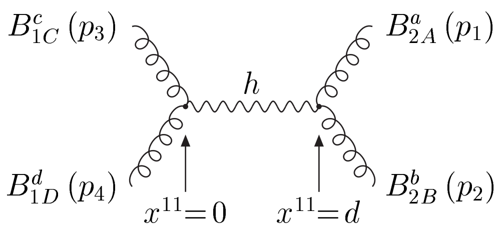


**Figure 1:** Graviton exchange.

Here we sum over all "colours" $a, b, c, d$ and physical polarizations $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ of the in- and out-states, since all of them add to the interaction of the two boundaries. For our conventions concerning annihilation and creation operators see appendix E. When in the next step we Wick-contract creation and annihilation operators $b^{a,\dagger}_{2\lambda}$ and $b^a_{1\lambda}$ with the boundary field operators $B^a_{iA}$, we have to take into account that creation and annihilation operators from the left-hand side of the diagram can only be contracted with left-hand sided $B^a_{1A}(x_1)$ operators and equally creation and annihilation operators from the right-hand side of the diagram can only be contracted with right-hand sided $B^a_{2A}(x_2)$ operators. If we would allow for "mixed" contractions, t- and u-channel diagrams would also be present. But these have to be excluded as they cannot arise when both hyperplanes do not coincide. After a further trivial integration over the circle coordinates, we are thus led to

$$
\begin{aligned}
S_h = & -\frac{1}{2}\,\kappa^2 \sum_{a,b,c,d}\ \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \int d^{10}x_1 \int d^{10}x_2 \times \\
& \times \Big[\underbrace{b^{d}_{1\lambda_4}(p_4)\,\partial_{A_1} B^{a_1}_{1B_1}}_{}(x_1)\,\underbrace{b^{c}_{1\lambda_3}(p_3)\,\partial_{C_1} B^{a_1}_{1D_1}}_{}(x_1) + \Big(b^{d}_{1\lambda_4}(p_4) \longleftrightarrow b^{c}_{1\lambda_3}(p_3)\Big)\Big] \times \\
& \times F^{A_1B_1C_1D_1E_1F_1}\, i\,\Delta_{E_1F_1,E_2F_2}(x_1 - x_2, -d)\, F^{A_2B_2C_2D_2E_2F_2} \times \\
& \times \Big[\partial_{A_2} \underbrace{B^{a_2}_{2B_2}(x_2)\, b^{b,\dagger}_{2\lambda_2}}_{}(p_2)\,\partial_{C_2} \underbrace{B^{a_2}_{2D_2}(x_2)\, b^{a,\dagger}_{2\lambda_1}}_{}(p_1) + \Big(b^{b,\dagger}_{2\lambda_2}(p_2) \longleftrightarrow b^{a,\dagger}_{2\lambda_1}(p_1)\Big)\Big]\,,
\end{aligned}
$$

where we have expressed the graviton 2-point function $\langle 0|T\left(h_{E_1F_1}(x_1,0)h_{E_2F_2}(x_2,d)\right)|0\rangle$ through $i$ times its propagator $\Delta_{M_1M_2,N_1N_2}(x_1 - x_2, -d)$. Using the expressions (E.7) and (E.8) for the Wick-contractions, gives the $E_8$ group factor $\sum_{a,b,c,d=1}^{248} \delta^{ab}\,\delta^{cd} = (248)^2$, and we arrive at the expression

$$
\begin{aligned}
S_h = & -i\,\frac{\kappa^2}{2}\,(248)^2 \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \int d^{10}x_1 \int d^{10}x_2\, e^{i(p_1+p_2)x_2}\, e^{-i(p_3+p_4)x_1} \times \\
& \times \Big[\epsilon_{B_1}(p_4,\lambda_4)\, p_{4,A_1}\epsilon_{D_1}(p_3,\lambda_3)\, p_{3,C_1} + \epsilon_{B_1}(p_3,\lambda_3)\, p_{3,A_1}\epsilon_{D_1}(p_4,\lambda_4)\, p_{4,C_1}\Big] \times \\
& \times F^{A_1B_1C_1D_1E_1F_1}\,\Delta_{E_1F_1,E_2F_2}(x_1 - x_2, -d)\, F^{A_2B_2C_2D_2E_2F_2} \times \\
& \times \Big[\epsilon_{B_2}(p_2,\lambda_2)\, p_{2,A_2}\epsilon_{D_2}(p_1,\lambda_1)\, p_{1,C_2} + \epsilon_{B_2}(p_1,\lambda_1)\, p_{1,A_2}\epsilon_{D_2}(p_2,\lambda_2)\, p_{2,C_2}\Big]\,.
\end{aligned}
$$

To utilize the previously derived flat-space propagator (2.3), we have to notice that the momentum in the compactified eleventh direction $p^{11}_m = m/R; m \in \mathbb{Z}$ is quantized. The radius $R$ of the circle is related via $R = d/\pi$ to the distance $d$ between the two hyperplanes. Ensuring that we do not change the dimensions of the propagator as compared to the flat case, we have to take

$$
f\left(x^{11}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dp^{11} e^{ip^{11}x^{11}} f\left(p^{11}\right) \sum_{m\in\mathbb{Z}} \delta\left(p^{11}d - m\pi\right) = \frac{1}{2\pi d}\sum_{m\in\mathbb{Z}} e^{ip^{11}_m x^{11}} f\left(p^{11}_m\right)
$$

as the Fourier transform in the eleventh direction. Therefore the Fourier-transformed graviton propagator reads

$$
\Delta_{E_1F_1,E_2F_2}\left(x_1 - x_2, x^{11}\right) = \frac{1}{d\,(2\pi)^{11}} \int d^{10}p\, e^{ip(x_1-x_2)} \sum_{m\in\mathbb{Z}} e^{ip^{11}_m x^{11}}\,\Delta_{E_1F_1,E_2F_2}\left(p, p^{11}_m\right),
$$

where $\Delta_{E_1F_1,E_2F_2}(p, p^{11}_m)$ is functionally the same as in the flat, non-compact case. Plugging

$$
\begin{aligned}
\Delta_{E_1F_1,E_2F_2}(x_1 - x_2, -d) = & \frac{1}{d\,(2\pi)^{11}} \int d^{10}p\, e^{ip(x_1-x_2)} \times \\
& \times \sum_{m\in\mathbb{Z}} (-1)^m\,\Delta_{E_1F_1,E_2F_2}\left(p, p^{11}_m\right). \qquad (3.2)
\end{aligned}
$$



JHEP05(2000)046

into the expression for $S_h$ and integrating over $x_1, x_2$, results in

$$
\begin{aligned}
S_h = & -i\,\frac{\kappa^2}{2d}\,(248)^2\,(2\pi)^9 \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \int d^{10}p\;\delta^{10}\left(p_1+p_2-p\right)\delta^{10}\left(-p+p_3+p_4\right)\times \\
& \times\Big[p_{4,A_1}\epsilon_{B_1}(p_4,\lambda_4)\,p_{3,C_1}\epsilon_{D_1}(p_3,\lambda_3)+p_{3,A_1}\epsilon_{B_1}(p_3,\lambda_3)\,p_{4,C_1}\epsilon_{D_1}(p_4,\lambda_4)\Big]\times \\
& \times F^{A_1B_1C_1D_1E_1F_1}\sum_{m\in\mathbb{Z}}(-1)^m\,\Delta_{E_1F_1,E_2F_2}\left(p,p^{11}_m\right)F^{A_2B_2C_2D_2E_2F_2}\times \\
& \times\Big[p_{2,A_2}\epsilon_{B_2}(p_2,\lambda_2)\,p_{1,C_2}\epsilon_{D_2}(p_1,\lambda_1)+p_{1,A_2}\epsilon_{B_2}(p_1,\lambda_1)\,p_{2,C_2}\epsilon_{D_2}(p_2,\lambda_2)\Big]\,.
\end{aligned}
$$

The integration over $p$ can now be trivially performed, resulting in an overall $D=10$ energy-momentum conserving delta-function. The interaction-amplitude or T-matrix element is defined by equating $S_h = i\,(2\pi)^{10}\,\delta^{10}(p_1+p_2-p_3-p_4)T_h$. Going in between to the center-of-mass (CMS) frame with respect to the ten-dimensional momenta parallel to the boundary, employing (2.3) plus various kinematical relations gathered in the appendix, we finally arrive at the expression

$$
T_h = \frac{2\kappa^2}{\pi d}\,(248)^2\left(25s^2-32tu\right)\sum_{m\in\mathbb{Z}}\frac{(-1)^m}{-s+\left(p^{11}_m\right)^2}\,.
$$

Here $s,t,u$ are the Mandelstam-variables composed out of the ten-dimensional parallel momenta, as pointed out earlier. To perform the sum, we use

$$
\sum_{m=1}^{\infty}\frac{(-1)^m}{z^2-m^2\pi^2} = \frac{1}{2z}\left(\frac{1}{\sin z}-\frac{1}{z}\right)\,,\qquad z\in\mathbb{C}\,,
$$

which one obtains as an application of the Mittag-Leffler theorem from Complex Analysis and get

$$
\sum_{m\in\mathbb{Z}}\frac{(-1)^m}{-s+\left(p^{11}_m\right)^2} = -\frac{d}{\sqrt{s}\sin\left(\sqrt{s}d\right)}\,. \tag{3.3}
$$

This yields for the matrix-element

$$
T_h(s,\vartheta) = -\frac{2\kappa^2}{\pi}\,(248)^2\,\frac{\left(25s^2-32tu\right)}{\sqrt{s}\sin\left(\sqrt{s}d\right)}\,. \tag{3.4}
$$

It is easy to verify that $25s^2-32tu>0$. Concerning our stability analysis, we should further integrate over the scattering angle $\vartheta$ of the CMS-system from 0 to $\pi/2$ (due to the fact that we have identical fields in the out state, the integration is only over half the usual range). This gives

$$
\mathcal{T}_h(s) = -21\,(248)^2\,\kappa^2\,\frac{s^{3/2}}{\sin\left(\sqrt{s}d\right)}\,. \tag{3.5}
$$

## 3.2 Gravitino exchange

There exist two diagrams describing amplitudes resulting from gravitino exchange. The first one is depicted in figure 2. Using (2.10) we obtain for its S-matrix

$$
\begin{aligned}
S_{\psi,I} = & -\frac{1}{2}\,\kappa^2 \sum_{a,b,c,d=1}^{248} \sum_{\lambda_2,\lambda_4=1}^{8} \sum_{s_1,s_3=1}^{8} \int d^{10}x_1 \int d^{10}x_2 \, \times \\
& \times \langle 0|\, b^d_{1\lambda_4}(p_4)\, d^c_{1s_3}(p_3)\, T\Bigg( :\frac{1}{2}\,\psi^{\alpha_1}_{A_1}(x_1,0) C_{\alpha_1\beta_1}\left(\Gamma^{B_1C_1}\Gamma^{A_1}\right)^{\beta_1}{}_{\gamma_1} \lambda_1^{a_1\gamma_1}(x_1)\partial_{B_1} \, \times \\
& \qquad \times B^{a_1}_{1C_1}(x_1) : \frac{1}{2}\,\psi^{\alpha_2}_{A_2}(x_2,d) C_{\alpha_2\beta_2}\left(\Gamma^{B_2C_2}\Gamma^{A_2}\right)^{\beta_2}{}_{\gamma_2} \, \times \\
& \qquad \times \lambda_2^{a_2\gamma_2}(x_2)\partial_{B_2} B^{a_2}_{2C_2}(x_2) : \Bigg) b^{b,\dagger}_{2\lambda_2}(p_2)\, d^{a,\dagger}_{2s_1}(p_1)\,|0\rangle \, .
\end{aligned}
$$

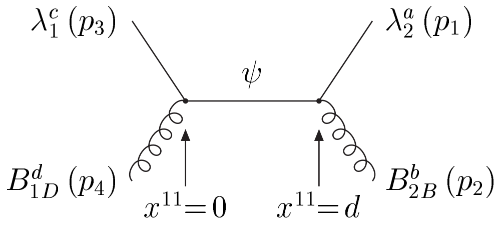


**Figure 2:** Gravitino exchange, I.

Again we sum over "colours" $a, b, c, d$, physical polarizations $\lambda_2, \lambda_4$ of the gauge fields and spin polarizations $s_1, s_3$ of the gauginos. $d^{a,\dagger}_{2s}$ and $d^a_{1s}$ are the creation and annihilation operators of the gauginos. Performing the Wick-contractions with the help of (E.7)–(E.10) and using the momentum representation (3.2) for the gravitino propagator (2.4) leads us to

$$
\begin{aligned}
S_{\psi,I} = & -\left(\frac{1}{2}\right)^2 \frac{\kappa^2}{2d}\,\frac{(248)^2}{(2\pi)^{11}} \, \times \\
& \times \sum_{\lambda_2,\lambda_4=1}^{8} \sum_{s_1,s_3=1}^{8} \int d^{10}p \sum_{m\in\mathbb{Z}} (-1)^m \int d^{10}x_1 \int d^{10}x_2 \, \times \\
& \times e^{-i(-p+p_3+p_4)x_1} e^{i(p_1+p_2-p)x_2}\, p_{4,B_1}\epsilon_{C_1}(p_4,\lambda_4)\bar{u}_{s_3,\gamma_2}(p_3)\left(\Gamma^{A_1}\Gamma^{B_1C_1}\right)^{\gamma_2}{}_{\alpha_1} \, \times \\
& \times \left(\tilde{\Delta}_{A_1A_2}\right)^{\alpha_1}{}_{\beta_2}\left(p, p^{11}_m\right)\left(\Gamma^{B_2C_2}\Gamma^{A_2}\right)^{\beta_2}{}_{\gamma_2} u^{\gamma_2}_{s_1}(p_1) p_{2,B_2}\epsilon_{C_2}(p_2,\lambda_2) \, .
\end{aligned}
$$

As before the factor of $(248)^2$ represents the $E_8$ group factor. Performing the $x_1, x_2$ integrations result in two Dirac delta functions describing the ten-dimensional energy-momentum conservation at each vertex separately. Upon integration over the momentum $p$, carried by the gravitino, we arrive at the following T-matrix element:

$$
\begin{aligned}
T_{\psi,I} = & \; i\,\frac{\kappa^2}{4\pi d}\left(\frac{1}{2}\right)^2 (248)^2 \, \times \\
& \times \sum_{\lambda_2,\lambda_4=1}^{8} \sum_{s_1,s_3=1}^{8} \sum_{m\in\mathbb{Z}} (-1)^m p_{4,B_1}\epsilon_{C_1}(p_4,\lambda_4)\bar{u}_{s_3,\gamma_2}(p_3)\left(\Gamma^{A_1}\Gamma^{B_1C_1}\right)^{\gamma_2}{}_{\alpha_1} \, \times \\
& \qquad \times \left(\tilde{\Delta}_{A_1A_2}\right)^{\alpha_1}{}_{\beta_2}\left(p_1+p_2, p^{11}_m\right)\left(\Gamma^{B_2C_2}\Gamma^{A_2}\right)^{\beta_2}{}_{\gamma_2} \, \times \\
& \qquad \times u^{\gamma_2}_{s_1}(p_1) p_{2,B_2}\epsilon_{C_2}(p_2,\lambda_2) \, .
\end{aligned}
$$

In order to facilitate this expression further, we note that the Weyl condition $\Gamma^{10}u_s(p) = u_s(p)$, $\bar{u}_s(p)\Gamma^{10} = -\bar{u}_s(p)$ for the gaugino spinor enforces $\bar{u}_s(p)\Gamma^{A_1}\dots\Gamma^{A_{2n}}u_{s'}(p') = 0$. Using this observation, the Weyl condition itself, the Dirac equation $\not{p}u_s(p) = \bar{u}_s(p)\not{p} = 0$ as well as the expression (2.5) for $\tilde{\Delta}_{A_1A_2}$, we receive in the CMS-frame

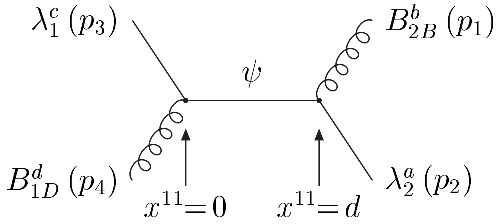


**Figure 3:** Gravitino exchange, II.

$$
\begin{aligned}
T_{\psi,I} = i\,\frac{\kappa^2}{4\pi d}\left(\frac{1}{2}\right)^2 (248)^2 \sum_{\lambda_2,\lambda_4=1}^{8}\,\sum_{s_1,s_3=1}^{8}\,\sum_{m\in\mathbb{Z}} \frac{(-1)^m}{-s+(p_m^{11})^2}\,\bar{u}_{s_3}(p_3)\,\times \\
\times\Bigg(E^2(4\cos\vartheta+28)\,\not{p}_4 + 2E\sin\vartheta\,\not{p}_4\,\not{\epsilon}(p_2,\lambda_2)\,\not{p}_2 - 2E^3\sin\vartheta\,\not{\epsilon}(p_4,\lambda_4) + \\
+\frac{2E^2}{3}\,\not{\epsilon}(p_4,\lambda_4)\,\not{p}_4\,\not{\epsilon}(p_2,\lambda_2) + \\
+E^2\left(\cos\vartheta - \frac{1}{3}\right)\,\not{\epsilon}(p_4,\lambda_4)\,\not{\epsilon}(p_2,\lambda_2)\,\not{p}_2\Bigg)u_{s_1}(p_1)\,.
\end{aligned}
$$

$E = \sqrt{s}$ and $\vartheta$ denote the ten-dimensional CMS-energy and the scattering angle in the CMS-frame along the hyperplanes (see B). Employing the explicit expressions for $\bar{u}_{s_3}(p_3)$, $u_{s_1}(p_1)$ and for the $\Gamma$-matrices from the appendix, we get

$$
T_{\psi,I} = -i\,\frac{\kappa^2}{4\pi d}\left(\frac{1}{2}\right)^2 (248)^2 \sum_{m\in\mathbb{Z}}(-1)^m\,128(s-u)\sqrt{\frac{-u}{s}}\,\frac{s}{-s+(p_m^{11})^2}\,.
$$

In addition to the diagram of figure 2, we also have to add the diagram of figure 3 which merely amounts to the exchange of $p_1 \leftrightarrow p_2$ or $t \leftrightarrow u$ in the preceding diagram. Adding up both contributions results in

$$
\begin{aligned}
T_\psi &= -i\,\frac{\kappa^2}{4\pi d}\left(\frac{1}{2}\right)^2 (248)^2\,\times \\
&\times\sum_{m\in\mathbb{Z}}(-1)^m 128\left((s-u)\sqrt{\frac{-u}{s}} + (s-t)\sqrt{\frac{-t}{s}}\right)\frac{s}{-s+(p_m^{11})^2}\,.
\end{aligned}
$$

If we utilize (3.3) again, we conclude

$$
T_\psi(s,\vartheta) = i\,\frac{8\kappa^2}{\pi}\,(248)^2\,\frac{\left((s-u)\,\sqrt{-u} + (s-t)\,\sqrt{-t}\right)}{\sin\left(\sqrt{s}d\right)}\,. \tag{3.6}
$$

For the stability analysis we perform a further integration over the scattering angle $\vartheta$ from 0 to $\pi$ (as appropriate for distinguishable fields in the out state), which finally gives

$$
\mathcal{T}_\psi(s) = i\,\frac{160\kappa^2}{3\pi}\,(248)^2\,\frac{s^{3/2}}{\sin\left(\sqrt{s}d\right)}\,.
$$

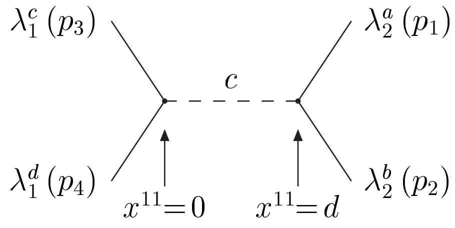


**Figure 4:** 3-form exchange.

### 3.3 3-form exchange

The 3-form exchange diagram of figure 4 yields the following expression for the S-matrix element:

$$
\begin{aligned}
S_c = -\frac{1}{2}\,\kappa^2 \sum_{a,b,c,d=1}^{248} \sum_{s_1,s_2,s_3,s_4=1}^{8} \int d^{10}x_1 \int d^{10}x_2 \;\times \\
\times \langle 0|\, d^{d}_{1s_4}(p_4)\, d^{c}_{1s_3}(p_3) T\Bigg( : \frac{1}{\sqrt{2}}\,\lambda_1^{a_1\alpha_1}(x_1) C_{\alpha_1\beta_1} (\Gamma^{A_1B_1C_1})^{\beta_1}{}_{\gamma_1} \lambda_1^{a_1\gamma_1}(x_1) \;\times \\
\times\; \partial_{[A_1} c_{B_1C_1 11]}(x_1,0) : \frac{1}{\sqrt{2}}\,\lambda_2^{a_2\alpha_2}(x_2) C_{\alpha_2\beta_2} (\Gamma^{A_2B_2C_2})^{\beta_2}{}_{\gamma_2} \;\times \\
\times\; \lambda_2^{a_2\,\gamma_2}(x_2) \partial_{[A_2} c_{B_2C_2 11]}(x_2,d) : \Bigg) d^{b,\dagger}_{2s_2}(p_2)\, d^{a,\dagger}_{2s_1}(p_1)\,|0\rangle\,.
\end{aligned}
$$

We make use of (E.9), (E.10) to perform the Wick-contractions and gain the $E_8$ gauge group factor $(248)^2$ as previously. We then combine the four resulting terms together by employing the relation $u^{\alpha}_{s}(p) C_{\alpha\beta} (\Gamma^{ABC})^{\beta}{}_{\gamma} u^{\gamma}_{s'}(p') \;=\; -u^{\alpha}_{s'}(p')\, C_{\alpha\beta} (\Gamma^{ABC})^{\beta}{}_{\gamma} u^{\gamma}_{s}(p)$. Moreover, expressing the 2-point function $\langle 0|T\,(c_{M_1M_2M_3}(x_1,0) c_{N_1N_2N_3}(x_2,d))\,|0\rangle$ as $i$ times the 3-form propagator $\Delta_{M_1M_2,N_1N_2}\,(x_1-x_2,-d)$ gives

$$
\begin{aligned}
S_c = -i\kappa^2 (248)^2 \sum_{s_1,s_2,s_3,s_4=1}^{8} \int d^{10}x_1 \int d^{10}x_2\, e^{-i(p_3+p_4)x_1}\, e^{i(p_1+p_2)x_2} \;\times \\
\times u^{\alpha_1}_{s_4}(p_4) C_{\alpha_1\beta_1} \left(\Gamma^{A_1B_1C_1}\right)^{\beta_1}{}_{\gamma_1} u^{\gamma_1}_{s_3}(p_3)\, \partial_{[A_1} \partial^{[A_2} \Delta_{B_1C_1 11]}{}^{B_2C_2 11]} \;\times \\
\times (x_1-x_2,-d) u^{\alpha_2}_{s_2}(p_2) C_{\alpha_2\beta_2} \left(\Gamma^{A_2B_2C_2}\right)^{\beta_2}{}_{\gamma_2} u^{\gamma_2}_{s_1}(p_1)\,.
\end{aligned}
$$

Fourier-transforming the propagator with the help of (2.6) and (3.2)

$$
\begin{aligned}
\partial_{[A_1} \partial^{[A_2} \Delta_{B_1C_1 11]}{}^{B_2C_2 11]}(x_1-x_2,-d) = \\
= -\frac{1}{d(3!)^2(2\pi)^{11}} \int d^{10}\,p e^{ip(x_1-x_2)} \sum_{m\in\mathbb{Z}} (-1)^m \left(\frac{3!}{4!}\right)^2 \times \\
\times \left( 3p_{[A_1} p^{[A_2}\, \delta^{B_2}_{B_1}\, \delta^{C_2]}_{C_1]} - \left(p^{11}_m\right)^2 \delta^{[A_2}_{[A_1}\, \delta^{B_2}_{B_1}\, \delta^{C_2]}_{C_1]} \right) \frac{1}{p^2 + (p^{11}_m)^2}
\end{aligned}
$$

brings us to

$$\begin{aligned} S_c = i\,\frac{\kappa^2}{d}\,\frac{(248)^2}{(4!)^2(2\pi)^{11}} \sum_{s_1,s_2,s_3,s_4=1}^{8} \int d^{10}p \sum_{m\in\mathbb{Z}} (-1)^m \int d^{10}x_1 \int d^{10}x_2\, e^{-i(-p+p_3+p_4)x_1} \times \\ \times\, e^{i(p_1+p_2-p)x_2} \Big(3u^{\alpha_1}_{s_4}(p_4)\, C_{\alpha_1\beta_1} \left(\Gamma^{A_1BC}\right)^{\beta_1}{}_{\gamma_1} u^{\gamma_1}_{s_3}(p_3)\, u^{\alpha_2}_{s_2}(p_2)\, C_{\alpha_2\beta_2} \left(\Gamma_{A_2BC}\right)^{\beta_2}{}_{\gamma_2} \times \\ \times\, u^{\gamma_2}_{s_1}(p_1)\, p_{A_1} p^{A_2} - u^{\alpha_1}_{s_4}(p_4)\, C_{\alpha_1\beta_1} \left(\Gamma^{ABC}\right)^{\beta_1}{}_{\gamma_1} u^{\gamma_1}_{s_3}(p_3)\, u^{\alpha_2}_{s_2}(p_2) \times \\ \times\, C_{\alpha_2\beta_2} \left(\Gamma_{ABC}\right)^{\beta_2}{}_{\gamma_2} u^{\gamma_2}_{s_1}(p_1) \left(p^{11}_m\right)^2 \Big) \frac{1}{p^2 + (p^{11}_m)^2}\,. \end{aligned}$$

After integration over $x_1, x_2$ and afterwards over $p$, we recognize the T-matrix element as

$$\begin{aligned} T_c = \frac{\kappa^2}{\pi d}\,\frac{(248)^2}{(3!)^3} \sum_{s_1,s_2,s_3,s_4=1}^{8} \sum_{m\in\mathbb{Z}} (-1)^m \times \\ \times \Big(3\bar{u}_{s_4}(p_4)\Gamma^{A_1BC}u_{s_3}(p_3)\bar{u}_{s_2}(p_2)\Gamma_{A_2BC}u_{s_1}(p_1)(p_3+p_4)_{A_1}(p_1+p_2)^{A_2} - \\ - \bar{u}_{s_4}(p_4)\Gamma^{ABC}u_{s_3}(p_3)\bar{u}_{s_2}(p_2)\Gamma_{ABC}u_{s_1}(p_1)\left(p^{11}_m\right)^2\Big) \frac{1}{-s + (p^{11}_m)^2}\,. \end{aligned}$$

In the following, we are dealing separately with the first and the second term of this amplitude in order to boil them down to some more enlightening expressions.

Let's start with the first term and the observation that the Dirac equation $\not{p}u_s(p) = \bar{u}(p)\not{p} = 0$ gives the relations

$$p_A\Gamma^{ABC}u_s(p) = 2p^{[B}\Gamma^{C]}u_s(p)\,, \qquad \bar{u}_s(p)\Gamma^{ABC}p_A = -2\bar{u}_s(p)\,p^{[B}\Gamma^{C]}\,.$$

If we apply them to the first term, we obtain

$$\begin{aligned} \sum_{s_1,s_2,s_3,s_4=1}^{8} 3\bar{u}_{s_4}(p_4)\Gamma^{A_1BC}u_{s_3}(p_3)\bar{u}_{s_2}(p_2)\Gamma_{A_2BC}u_{s_1}(p_1)(p_3+p_4)_{A_1}(p_1+p_2)^{A_2} = \\ = 12 \sum_{s_1,s_2,s_3,s_4=1}^{8} \bar{u}_{s_4}(p_4)(p_3-p_4)^{[B}\Gamma^{C]}u_{s_3}(p_3)\bar{u}_{s_2}(p_2)(p_1-p_2)_{[B}\Gamma_{C]}u_{s_1}(p_1)\,. \end{aligned}$$

By noticing that in the CMS-frame $p_1 - p_2 = (0, \ldots, 0, E)$ and $p_3 - p_4 = (0, \ldots, 0, E\sin\vartheta, E\cos\vartheta)$, we eventually reduce this expression to

$$\begin{aligned} 4!E^2 \sum_{s_1,s_2,s_3,s_4=1}^{8} \Big(\cos\vartheta\bar{u}_{s_4}(p_4)\Gamma^9u_{s_3}(p_3)\bar{u}_{s_2}(p_2)\Gamma_9u_{s_1}(p_1) + \\ + \sin\vartheta\bar{u}_{s_4}(p_4)\Gamma^9u_{s_3}(p_3)\bar{u}_{s_2}(p_2)\Gamma_8u_{s_1}(p_1) - \\ - \cos\vartheta\bar{u}_{s_4}(p_4)\Gamma^Au_{s_3}(p_3)\bar{u}_{s_2}(p_2)\Gamma_Au_{s_1}(p_1)\Big) = -4! \times 64s^2\,. \end{aligned}$$

Concerning the second part of the amplitude, we decompose

$$\Gamma^{ABC} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

into $2 \times 2$ and $16 \times 16$ matrices. With the explicit expression for the gaugino-spinor (D.1), we find

$$\begin{aligned} \bar{u}_{s_4}(p_4)\Gamma^{ABC}u_{s_3}(p_3) &= Ec\left(\frac{\sin\vartheta}{2}A + \cos^2\frac{\vartheta}{2}\,B - \sin^2\frac{\vartheta}{2}\,C - \frac{\sin\vartheta}{2}\,D\right)_{s_4s_3}, \\ \bar{u}_{s_2}(p_2)\Gamma^{ABC}u_{s_1}(p_1) &= EcB_{s_2s_1}\,. \end{aligned}$$

In particular, we have for our chosen representation (given in D) of $SO(1,10)$ $\Gamma$-matrices the following description in terms of $8 \times 8$ submatrices $\gamma^a$

$$\begin{aligned} &\Gamma^{0ab}:\ c = 1\,, \qquad B = 0\,, && \Gamma^{0a9}:\ c = 1\,, \qquad B = \gamma^a\,, \\ &\Gamma^{abc}:\ c = 1\,, \qquad B = \gamma^a\gamma^{b,T}\gamma^c\,, && \Gamma^{ab9}:\ c = 1\,, \qquad B = 0\,. \end{aligned}$$

Since after summation over the spin-polarizations, an antisymmetric matrix $B$ gives a vanishing contribution, the only non-vanishing terms for our scattering process stem from $\Gamma^{089}$ and $\Gamma^{ijk}$; $i,j,k = 1,\ldots,7$. Hence we are able to reduce the second term to

$$\begin{aligned} \sum_{s_1,s_2,s_3,s_4=1}^{8} &\bar{u}_{s_4}(p_4)\Gamma^{ABC}u_{s_3}(p_3)\bar{u}_{s_2}(p_2)\Gamma_{ABC}\,u_{s_1}(p_1) = \\ &= 3! \times E^2\left(-64 + \sum_{i<j<k}\underbrace{\left[\sum_{s_2,s_1}\left(\gamma^{ijk}\right)_{s_2s_1}\right]^2}_{(\pm 8)^2}\right) = (3! \times 8E)^2\,. \end{aligned}$$

Putting the results for the two terms together, we arrive at the following expression for the 3-form exchange amplitude

$$T_c = -\frac{\kappa^2}{\pi d}\,\frac{128(248)^2}{(3!)^2}\,s\sum_{m\in\mathbb{Z}}(-1)^m\,\frac{2s + 3(p_m^{11})^2}{-s + (p_m^{11})^2}\,. \tag{3.7}$$

Using again (3.3) for the summation, we find

$$\sum_{m\in\mathbb{Z}}(-1)^m\,\frac{2s + 3(p_m^{11})^2}{-s + (p_m^{11})^2} = 3\sum_{m\in\mathbb{Z}}(-1)^m - 5\,\frac{\sqrt{s}d}{\sin\left(\sqrt{s}d\right)}\,.$$

The first term describes an alternating sum, which does not converge and requires some kind of regularization. In order to understand this contribution, we will explore in a moment the $d \to \infty$ limit. Therefore it proves useful to express the above obtained amplitude in terms of the $D = 10$ gravitational coupling constant $\kappa_{(10)}$ which is independent of the compactification radius $R$ or $d$. Compactification of M-theory on an $S^1$ of radius $R$ and a subsequent comparison of its Einstein-Hilbert term with the Einstein-Hilbert term coming from the effective action of the $D = 10$ heterotic string (in Einstein frame) leads to the following relationship between the $D = 11$ and the $D = 10$ gravitational coupling constants

$$\kappa^2 = 2d\kappa^2_{(10)}\,. \tag{3.8}$$

Hence $T_c$ can be expressed as

$$T_c = -\kappa^2_{(10)}\,\frac{256(248)^2}{(3!)^2\pi}\,s\left(3\sum_{m\in\mathbb{Z}}(-1)^m - 5\frac{\sqrt{s}d}{\sin\left(\sqrt{s}d\right)}\right).$$

Since the first part of the amplitude, consisting of the alternating sum and some $d$-independent prefactors, is independent of $d$, we can equally well evaluate it at any $d$, in particular at $d \to \infty$. Secondly, if we consider a large radius, the difference between two adjacent values of $p^{11}_m$ becomes infinitesimally small and we are allowed to replace the sum by an integral

$$\lim_{d\to\infty}\sum_{m\in\mathbb{Z}} f\left(p^{11}_m = \frac{m}{R} = m\,\frac{\pi}{d}\right) = \lim_{d\to\infty}\frac{d}{\pi}\int_{-\infty}^{\infty} dp^{11} f\left(p^{11}\right).$$

Writing $(-1)^m = e^{ip^{11}_m d}$, we now encounter the following expression for the alternating sum

$$\sum_{m\in\mathbb{Z}}(-1)^m = \sum_{m\in\mathbb{Z}} e^{ip^{11}_m d} = \lim_{d\to\infty}\frac{d}{\pi}\int_{-\infty}^{\infty} dp^{11} e^{ip^{11}d} = \lim_{d\to\infty} 2\,d\,\delta(d) = 0\,.$$

Thus finally the amplitude can be completely determined to be

$$T_c(s) = \frac{\kappa^2}{\pi}\,\frac{160(248)^2}{9}\,\frac{s^{3/2}}{\sin\left(\sqrt{s}d\right)}\,. \tag{3.9}$$

The integration over the scattering angle from 0 to $\pi/2$ is trivial and results in

$$\mathcal{T}_c(s) = \kappa^2\,\frac{80(248)^2}{9}\frac{s^{3/2}}{\sin\left(\sqrt{s}d\right)}\,. \tag{3.10}$$

### 3.4 Two further Graviton exchange diagrams

To complete our discussion of all relevant tree diagrams, which contribute to a boundary-boundary interaction, we also have to consider two further graviton exchange diagrams, depicted in figure 5. Both are, after performing the Wick-contractions, proportional to

$$-\bar{u}_{s_2}(p_2)\left(\Gamma^C\eta^{AB} - \Gamma^B\eta^{AC}\right)u_{s_1}(p_1) + \bar{u}_{s_1}(p_1)\left(\Gamma^C\eta^{AB} - \Gamma^B\eta^{AC}\right)u_{s_2}(p_2)\,,$$

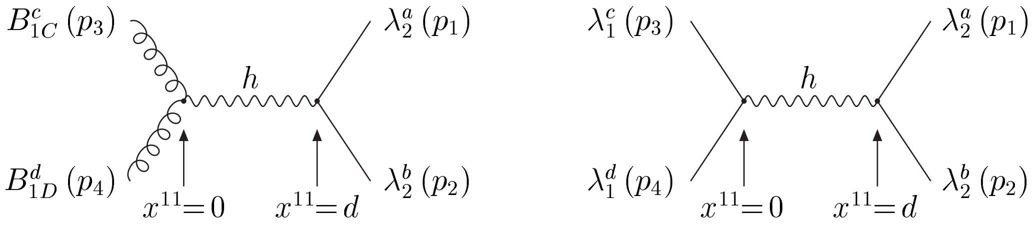


**Figure 5:** Vanishing Graviton exchange diagrams.

which gives zero, if we do avail ourselves of (D.2). Physically the vanishing of the diagrams is clear, since interchanging the two gauginos of the final state gives a minus-sign, which cannot be compensated for by the coupling to a graviton. In the previously analysed case of the coupling between two gauginos and the 3-form potential, the coupling delivers an extra minus-sign under exchange of the two fermions, so that the amplitude did not vanish in that case.

### 3.5 Analysis of the amplitudes

Gathering all the obtained amplitudes, integrated over the scattering angle, we have

$$\mathcal{T}_h(s) = -21(248)^2\,\kappa^2\,\frac{s^{3/2}}{\sin(\sqrt{s}d)}\,, \tag{3.11}$$

$$\mathcal{T}_\psi(s) = i\,\frac{160}{3\pi}\,(248)^2\,\kappa^2\frac{s^{3/2}}{\sin(\sqrt{s}d)}\,, \tag{3.12}$$

$$\mathcal{T}_c(s) = \frac{80}{9}\,(248)^2\,\kappa^2\frac{s^{3/2}}{\sin(\sqrt{s}d)}\,. \tag{3.13}$$

First of all, we have to determine the range of validity of (3.11)–(3.13). From the denominator we recognize that singularities occur at the excitations of the Kaluza-Klein states at $\sqrt{s} = m\pi/d = p^{11}_m\,;\ m \in \mathbb{Z}$. Our analysis did not cover contributions to the interaction amplitudes coming from these states and only included the exchange of the massless supergravity multiplet. Therefore, the range of validity of our results is subjected to the following constraint, given by the first Kaluza-Klein excitation

$$0 \le \sqrt{s} < \frac{\pi}{d}\,.$$

In the special case of vanishing CMS-energy $\sqrt{s} = 0$, each amplitude vanishes separately. This corresponds to the situation where the boundary fields on each fixed plane run in parallel directions. In this case we have trivially no interaction between the two boundaries, as expected from the vanishing of the susy-variations for this kinematics. In this special situation the flat background with vanishing $G$-flux corresponds to a stable ground state of the heterotic M-theory set-up. However, if there are excitations on the boundary, by which we mean a kinematical situation showing $\sqrt{s} > 0$ for the boundary-fields, we see that pure gravity leads to an attraction (since we are always below the first Kaluza-Klein excitation energy), whereas — similar to the behaviour of the RR-forms in the analogous D-brane case of type-II string theory — the 3-form exchange leads to a repulsion. If we choose the same CMS-energy for all three contributions, then the attractive gravity dominates the weaker 3-form repulsion. Hence the real part of the amplitudes indicates an instability which is caused by an attractive force trying to bring the two-boundaries closer together.

Thus the flat background with vanishing $G$-flux does not represent a stable vacuum in the presence of arbitrary momenta of the boundary-fields. An obvious guess as to the nature of a stable vacuum comes from the treatment of heterotic M-theory compactified on a Calabi-Yau threefold [3]. There it has been shown, that with a non-vanishing $G$-flux on the Calabi-Yau and the orbifold-direction, compactified heterotic M-theory exhibits a warped-geometry. In view of the failure of the flat vacuum to represent a stable configuration, one would naively think, that the warping of the geometry should survive in the decompactification limit. Ten-dimensional Poincaré-invariance indeed only allows for a non-trivial dependence on $x^{11}$ and hence a warped-geometry. However, the very Poincaré-invariance also requires $G_{KLMN}$ to vanish and therefore other sources for a warping of space-time must be taken into account.

The behaviour of the above calculated amplitudes is similar to the weakly-coupled string-theory case in which an excited D-brane can decay into a massless closed string state and the non-excited D-brane [11]. Such a decay is also possible whenever the two massless waves on the D-brane run in different directions and accordingly possess $\sqrt{s} > 0$. If the massless waves, however, run in the same direction, i.e. have $\sqrt{s} = 0$, then one is dealing with a BPS state which does not decay.

Curiously the gravitino exchange gives rise to an imaginary part. By inspection of (3.6) we find that the forward scattering amplitude $T_\psi(s, \vartheta = 0)$ is non vanishing. Via the optical theorem this would signal the opening of some inelastic channels for a decay of an excited boundary and therefore an instability in a more drastic sense.

A last remark concerns unitarity. If we would evaluate total cross-sections with the above amplitudes, then by integrating over the appropriate phase space, we would get at high energies

$$\sigma \sim |\mathcal{T}(s)|^2 s^2 \sim \kappa^4 s^5 \,.$$

However, unitarity of the S-matrix would lead for spinless states to the following restriction on partial wave amplitudes

$$\sigma_J \leq \frac{P_J}{s^4} \,,$$

where $P_J$ is some polynomial in $J$ independent of $s$. Neglecting $\vartheta$-dependent factors which arise for states with higher spin, we conclude, that the total cross-section $\sigma$ which is the sum of all $\sigma_J$, should decrease with increasing energy in order to obey unitarity. Since our cross-sections increase with energy, they violate unitarity. This is also plausible from the fact, that Hořava-Witten supergravity is not gauge invariant at the classical level and therefore no Ward-identities guarantee unitarity. However, we have to keep in mind the restriction to the energy regime $\sqrt{s} < \pi/d$ of our analysis. Should it happen, that a violation of unitarity occurs at an energy much higher than $\pi/d$, we would have to include the effects of the Kaluza-Klein excitations to decide, whether unitarity is violated or obeyed.

## 4. Comparison of the interaction amplitudes with the weakly coupled heterotic string amplitudes

According to the conjecture made in [1], we should recover the $D = 10$ weakly coupled heterotic $E_8 \times E_8$ string theory in the limit of small $R$ resp. $d$. Since the amplitudes which we have derived, so far, describe the low-energy regime, we should also compare to the analogous low-energy string amplitudes. Here we have to use the expressions (3.4), (3.6) and (3.9) which contain the full angular information. In order to derive the zero radius limit, we express all the derived amplitudes via (3.8) through the radius-independent $\kappa_{(10)}$ and then perform the limit $\sqrt{s}d \to 0$

$$T_h(s,\vartheta) = -\frac{4\kappa_{(10)}^2}{\pi}\,(248)^2\,\frac{(25s^2 - 32tu)d}{\sqrt{s}\sin(\sqrt{s}d)}$$

$$\xrightarrow{\sqrt{s}d\to 0} -\kappa_{(10)}^2\,\frac{4(248)^2}{\pi}\left(25\,s - 32\,\frac{tu}{s}\right), \tag{4.1}$$

$$T_\psi(s,\vartheta) = i\,\frac{16\kappa_{(10)}^2}{\pi}\,(248)^2\,\frac{\left((s-t)\sqrt{-t} + (s-u)\sqrt{-u}\right)d}{\sin(\sqrt{s}d)} \tag{4.2}$$

$$\xrightarrow{\sqrt{s}d\to 0} i\,\frac{16\kappa_{(10)}^2}{\pi}(248)^2\left((s-t)\sqrt{-\frac{t}{s}} + (s-u)\sqrt{-\frac{u}{s}}\right), \tag{4.3}$$

$$T_c(s) = \frac{320\kappa_{(10)}^2}{9\pi}\,(248)^2\frac{s^{3/2}d}{\sin(\sqrt{s}d)} \xrightarrow{\sqrt{s}d\to 0} \kappa_{(10)}^2\frac{320(248)^2}{9\pi}\,s\,. \tag{4.4}$$

So far for the M-theory amplitudes. Closed string amplitudes involve a factor $\kappa_{(10)}^{M-2+2L}$, where $M$ is the number of external particles and $L$ the number of loops. Hence with four external particles it is clear, that a factor $\kappa_{(10)}^2$ corresponds to string tree-amplitudes as well. Those heterotic string tree-amplitudes can be found in [12].[9] The terms, which originate there from taking traces of four $E_8 \times E_8$ group generators $T_i$, must be discarded from our comparison, since they correspond to processes where super Yang-Mills fields are exchanged between the initial and final states. What we want instead to compare to are the amplitudes which are generated by the exchange of states of the supergravity multiplet. Since they comprise singlet-representations under the $E_8 \times E_8$ gauge group, we merely encounter terms with traces over two generators, respectively. The string-theoretic tree-amplitudes adapted to our conventions read

$$A = \kappa_{10}^2 K\left(\zeta_1, \frac{p_1}{2}, \zeta_2, \frac{p_2}{2}, \zeta_3, \frac{p_3}{2}, \zeta_4, \frac{p_4}{2}\right) C\left(s,t,u\right) G\left(p_1,p_2,p_3,p_4,T_1,T_2,T_3,T_4\right),$$

where

$$\begin{aligned} C\left(s,t,u\right) &= -\pi\,\frac{\Gamma(-s/8)\Gamma(-t/8)\Gamma(-u/8)}{\Gamma(1+\frac{s}{8})\Gamma(1+\frac{t}{8})\Gamma(1+\frac{u}{8})}\,, \\ G\left(p_1,p_2,p_3,p_4,T_1,T_2,T_3,T_4\right) &= \frac{1}{32}\left(\frac{tu}{1+\frac{s}{8}}\,\mathrm{tr}\left[T_1T_2\right]\mathrm{tr}\left[T_3T_4\right]\right). \end{aligned}$$

The factor $G$ of [12] also contains terms describing a t- and a u-channel exchange. Since in the heterotic M-theory calculation for finite $d$, we get only s-channel contributions for interactions of the boundary fields via bulk fields, our expressions for $d \to 0$ should only be compared to this very s-channel part of the string calculation. For this reason we have omitted the t- and u-contributions to the $G$-factor. The generator $T_i$ corresponds to the $i$-th external particle and tr is defined as the trace in the adjoint representation of $E_8 \times E_8$ divided by 30. The various $\zeta_i$ stand for the polarization of the $i$-th particle. If it is a spinor, we have to substitute $\zeta_i = u_{s_i}(p_i)$, whereas for a gauge boson we have to take its polarization $\zeta_i = \epsilon_i(p_i, \lambda_i)$.

The $K$-factor describes the kinematics of the interaction and is given for the various cases[10] by

[9] The translation between the momenta $k_i^{GSW}$ and Mandelstam-variables used by [12] and the $p_i$ used in this paper is given by

$$\begin{aligned} k_1^{GSW} &= p_1\,, \qquad k_2^{GSW} = p_2\,, \qquad k_3^{GSW} = -p_3\,, \qquad k_4^{GSW} = -p_4\,, \\ s^{GSW} &= -\left(k_1^{GSW}+k_2^{GSW}\right)^2 = -(p_1+p_2)^2 = s\,, \\ t^{GSW} &= -\left(k_1^{GSW}+k_4^{GSW}\right)^2 = -(p_1-p_4)^2 = u\,, \\ u^{GSW} &= -\left(k_1^{GSW}+k_3^{GSW}\right)^2 = -(p_1-p_3)^2 = t\,. \end{aligned}$$

[10] There is no factor $K(u_1,u_2,\epsilon_3,\epsilon_4)$, since as in the heterotic M-theory calculation the $BB \to \lambda\lambda$ contribution vanishes.

$$
\begin{aligned}
&K\left(\epsilon_1,\frac{p_1}{2},\epsilon_2,\frac{p_2}{2},\epsilon_3,\frac{p_3}{2},\epsilon_4,\frac{p_4}{2}\right)= \\
&=\frac{1}{2^4}\Bigg(-\frac{1}{4}\left(su\,\epsilon_1{\cdot}\epsilon_3\epsilon_2{\cdot}\epsilon_4+st\epsilon_2{\cdot}\epsilon_3\epsilon_1{\cdot}\epsilon_4+tu\,\epsilon_1{\cdot}\epsilon_2\epsilon_3{\cdot}\epsilon_4\right)- \\
&\qquad-\frac{s}{2}\left(\epsilon_1{\cdot}p_4\epsilon_3{\cdot}p_2\epsilon_2{\cdot}\epsilon_4+\epsilon_2{\cdot}p_3\epsilon_4{\cdot}p_1\epsilon_1{\cdot}\epsilon_3+\epsilon_1{\cdot}p_3\epsilon_4{\cdot}p_2\epsilon_2{\cdot}\epsilon_3+\epsilon_2{\cdot}p_4\epsilon_3{\cdot}p_1\epsilon_1{\cdot}\epsilon_4\right)+ \\
&\qquad+\frac{t}{2}\left(-\epsilon_1{\cdot}p_2\epsilon_4{\cdot}p_3\epsilon_3{\cdot}\epsilon_2-\epsilon_3{\cdot}p_4\epsilon_2{\cdot}p_1\epsilon_1{\cdot}\epsilon_4+\epsilon_1{\cdot}p_4\epsilon_2{\cdot}p_3\epsilon_3{\cdot}\epsilon_4+\epsilon_3{\cdot}p_2\epsilon_4{\cdot}p_1\epsilon_1{\cdot}\epsilon_2\right)+ \\
&\qquad+\frac{u}{2}\left(-\epsilon_2{\cdot}p_1\epsilon_4{\cdot}p_3\epsilon_3{\cdot}\epsilon_1-\epsilon_3{\cdot}p_4\epsilon_1{\cdot}p_2\epsilon_2{\cdot}\epsilon_4+\epsilon_2{\cdot}p_4\epsilon_1{\cdot}p_3\epsilon_3{\cdot}\epsilon_4+\epsilon_3{\cdot}p_1\epsilon_4{\cdot}p_2\epsilon_2{\cdot}\epsilon_1\right)\Bigg)\,, \\
&K\left(u_1,\frac{p_1}{2},\epsilon_2,\frac{p_2}{2},u_3,\frac{p_3}{2},\epsilon_4,\frac{p_4}{2}\right)=\frac{1}{2^3}\Bigg(-\frac{u}{2}\,\bar u_1\;\not\epsilon_2\left(\not p_3+\not p_4\right)\not\epsilon_4u_3+ \\
&\qquad\qquad\qquad\qquad\qquad\qquad+\frac{s}{2}\,\bar u_1\;\not\epsilon_4\left(\not p_2-\not p_3\right)\not\epsilon_2u_3\Bigg)\,, \\
&K\left(\epsilon_1,\frac{p_1}{2},u_2,\frac{p_2}{2},u_3,\frac{p_3}{2},\epsilon_4,\frac{p_4}{2}\right)=K\left(u_2,\frac{p_2}{2},\epsilon_1,\frac{p_1}{2},\epsilon_4,\frac{p_4}{2},u_3,\frac{p_3}{2}\right)= \\
&=\;\frac{1}{2^3}\left(\frac{u}{2}\bar u_2\;\not\epsilon_1\left(\not p_3+\not p_4\right)\not\epsilon_4u_3+s\left(\bar u_2\;\not\epsilon_4u_3p_4{\cdot}\epsilon_1+\bar u_2\;\not\epsilon_1u_3p_1{\cdot}\epsilon_4-\bar u_2\;\not p_4u_3\epsilon_1{\cdot}\epsilon_4\right)\right)\,, \\
&K\left(u_1,\frac{p_1}{2},u_2,\frac{p_2}{2},u_3,\frac{p_3}{2},u_4,\frac{p_4}{2}\right)=\frac{1}{2^2}\left(-\frac{s}{2}\,\bar u_2\Gamma^Au_3\bar u_1\Gamma_Au_4+\frac{u}{2}\bar u_1\Gamma^Au_2\bar u_4\Gamma_Au_3\right).
\end{aligned}
$$

Summing over every occuring vectorial or spinorial polarization index, we can simplify the kinematical factors further to

$$
\begin{aligned}
\sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}K\left(\epsilon_1,\frac{p_1}{2},\epsilon_2,\frac{p_2}{2},\epsilon_3,\frac{p_3}{2},\epsilon_4,\frac{p_4}{2}\right)&=-\frac{1}{8}\left(st+tu+us\right)+\frac{7}{16}\left(s^2+t^2+u^2\right), \\
\sum_{s_1,\lambda_2,s_3,\lambda_4}K\left(u_1,\frac{p_1}{2},\epsilon_2,\frac{p_2}{2},u_3,\frac{p_3}{2},\epsilon_4,\frac{p_4}{2}\right)&=4s(s-u)\sqrt{-\frac{u}{s}}\,, \\
\sum_{\lambda_1,s_2,s_3,\lambda_4}K\left(\epsilon_1,\frac{p_1}{2},u_2,\frac{p_2}{2},u_3,\frac{p_3}{2},\epsilon_4,\frac{p_4}{2}\right)&=4s(s-t)\sqrt{-\frac{t}{s}}\,, \\
\sum_{s_1,s_2,s_3,s_4}K\left(u_1,\frac{p_1}{2},u_2,\frac{p_2}{2},u_3,\frac{p_3}{2},u_4,\frac{p_4}{2}\right)&=-4\left(3s^2+(t-u)^2\right).
\end{aligned}
$$

In the low-energy limit $\alpha' s, \alpha' t, \alpha' u \to 0$ we have

$$
C\left(s,t,u\right)\longrightarrow\frac{2^9\pi}{stu}
$$

such that finally we arrive at the following expressions for the low-energy limit of the heterotic string amplitudes:

$$
A_{BB\overset{s}{\to}BB}=\pi\kappa_{10}^2 16\left(\mathrm{tr}\left[T_1T_2\right]\right)^2\left(s-\frac{tu}{s}\right), \tag{4.5}
$$

$$
\begin{aligned}
A_{\lambda B\overset{s}{\to}\lambda B}+A_{\lambda B\overset{s}{\to}B\lambda}&=\pi\kappa_{10}^2 4\times 16\left(\mathrm{tr}\left[T_1T_2\right]\right)^2\times \\
&\times\left((s-t)\sqrt{-\frac{t}{s}}+(s-u)\sqrt{-\frac{u}{s}}\right),
\end{aligned} \tag{4.6}
$$

$$A_{\lambda\lambda \xrightarrow{s} \lambda\lambda} = -\pi\kappa_{10}^2 (16)^2 \left(\mathrm{tr}\,[T_1 T_2]\right)^2 \left(s - \frac{tu}{s}\right). \tag{4.7}$$

If we compare these with (4.1), (4.3) and (4.4), we recognize substantial differences. Whereas (4.1) and (4.5) deviate mildly in their functional dependence on $s, t, u$, the discrepancy between (4.4) and (4.7) is manifest. The string amplitude shows an angular dependence but the heterotic M-theory amplitude is isotropic. The gravitino exchange amplitudes agree in their angular dependence. Nevertheless the string amplitude is real, whereas its M-theoretic counterpart is purely imaginary.

However, such a disagreement had to be expected. Generally, a low-energy description in terms of effective supergravity is only valid at large distances resp. small curvatures. Furthermore, Hořava-Witten supergravity is organized as a long wavelength expansion in the parameter $\kappa^{2/3}$, assumed to be small as compared to the eleventh-dimensional Planck-scale. However, in the limit $d \to 0$ of coinciding boundaries, the long-wavelength supergravity approximation breaks down and one cannot trust the order $\kappa^{2/3}$ expansion any longer. Therefore the effective description gives incorrect results.

## 5. Summary and conclusion

Our aim in this paper has been to vet whether heterotic M-theory on a flat background in its concrete low-energy formulation as Hořava-Witten supergravity is stable. Therefore, we used the background field method to derive interaction amplitudes between the fields living on the boundaries. Each amplitude vanishes trivially for non-excited boundaries which corresponds to a BPS ground state configuration as in the similar D-brane case. Nevertheless for a gauge excitation on the boundary with a CMS-energy $0 < \sqrt{s} < \pi/d$ below the first Kaluza-Klein resonance, the gravitational attraction dominated the 3-form repulsion, thereby giving rise to a net attractive force between the two boundaries. If one included also the contribution from the gravitino exchange, this generated an imaginary part for the amplitude. Via the optical theorem, an even more drastic instability of the effective theory was signalled. The consequence would be an inelastic decay of the flat vacuum set-up towards an energetically more favoured configuration. To avoid this instability, we argued, that already for the uncompactified heterotic M-theory the vacuum should exhibit a warped-geometry.

We briefly indicated that unitarity seems to be violated by the interaction amplitudes, which is also understandable from the fact, that the effective Hořava-Witten description is classically not gauge invariant. Since our treatment was reliable only for energies below the first Kaluza-Klein resonance, a seeming violation of unitarity above that limit could not be tested.

Finally, we extrapolated the M-theory amplitudes to the extreme limit in which both boundaries coincide. According to the conjecture, this is the realm where we



JHEP05(2000)046

should recover the $E_8 \times E_8$ heterotic string. But a comparison with the low-energy heterotic string amplitudes showed no complete agreement. However, this discrepancy had to be expected, since in this limit the effective supergravity description and in particular the long wavelength expansion of Hořava-Witten supergravity breaks down and leads to false results.


## Acknowledgments

We would like to thank I. Antoniadis, M. Faux, P. Hořava, and D. Lüst for helpful comments or remarks on partial aspects of this work.


## A. Notations

**Tensors.**

$$A_{(M_1 \dots M_n)} = \frac{1}{n!} \left( A_{M_1 \dots M_n} \pm (n! - 1) \text{ symmetric permutations} \right), \tag{A.1}$$

$$A_{[M_1 \dots M_n]} = \frac{1}{n!} \left( A_{M_1 \dots M_n} \pm (n! - 1) \text{ anti-symmetric permutations} \right). \tag{A.2}$$

## B. Mandelstam-variables and kinematic for the CMS

**Center-of-Mass variables:**

$$\text{scattering angle: } 0 \leq \vartheta \leq \pi \,, \qquad \text{CMS-Energy: } E \,. \tag{B.1}$$

Without loss of generality we can arrange the scattering such that the two incoming fields with momenta $p_1, p_2$ collide head-on in the CMS-system in the direction of the 9-th coordinate axis and the outgoing fields move on the plane spanned by the 8-th and 9-th coordinate axes. We choose $\vartheta$ to be the angle between $p_1$ and $p_3$. Concretely we take for the $D = 10$ momenta

$$\begin{aligned}
p_1 &= \left( \frac{E}{2}, 0, \dots, 0, \frac{E}{2} \right), \qquad p_2 = \left( \frac{E}{2}, 0, \dots, 0, -\frac{E}{2} \right), \\
p_3 &= \left( \frac{E}{2}, 0, \dots, 0, \frac{E}{2} \sin\vartheta, \frac{E}{2} \cos\vartheta \right), \\
p_4 &= \left( \frac{E}{2}, 0, \dots, 0, -\frac{E}{2} \sin\vartheta, -\frac{E}{2} \cos\vartheta \right).
\end{aligned}$$

**Mandelstam-variables:**

$$\begin{aligned}
s &= -\left( p_1 + p_2 \right)^2 = -\left( p_3 + p_4 \right)^2 = E^2 \,, \\
t &= -\left( p_1 - p_3 \right)^2 = -\left( p_2 - p_4 \right)^2 = -\frac{s}{2} \left[ 1 - \cos\vartheta \right] = -s \sin^2 \frac{\vartheta}{2} \leq 0 \,, \\
u &= -\left( p_1 - p_4 \right)^2 = -\left( p_2 - p_3 \right)^2 = -\frac{s}{2} \left[ 1 + \cos\vartheta \right] = -s \cos^2 \frac{\vartheta}{2} \leq 0 \,, \\
& \qquad s + t + u = 0 \,.
\end{aligned}$$

## C. $D = 10$ polarization vectors

**CMS-Polarization vectors.** Let the $D = 10$ momentum in the CMS be

$$p = \frac{E}{2}(1, \ldots, \sin\vartheta, \cos\vartheta)\,. \tag{C.1}$$

Then we have 8 transverse polarizations, which are given in the CMS by the following real vectors

$$\epsilon(p,1) = (0,1,0,\ldots,0)\,, \tag{C.2}$$
$$\epsilon(p,2) = (0,0,1,\ldots,0)\,, \tag{C.3}$$
$$\vdots \tag{C.4}$$
$$\epsilon(p,7) = (0,0,0,\ldots,1,0,0)\,, \tag{C.5}$$
$$\epsilon(p,8) = (0,0,0,\ldots,0,\cos\vartheta,-\sin\vartheta)\,. \tag{C.6}$$

**Useful contractions.** If we sum over all polarizations, we get

$$\sum_{\lambda=1}^{8} p_{1,A}\epsilon^A(p_2,\lambda) = \frac{E}{2}\sin(\vartheta_1 - \vartheta_2)\,. \tag{C.7}$$

Contractions of two polarization vectors are given by

$$\sum_{\lambda,\tilde{\lambda}=1}^{8} \epsilon_A(p,\lambda)\,\epsilon^A(\tilde{p},\tilde{\lambda}) = 7 + \cos(\vartheta - \tilde{\vartheta})\,. \tag{C.8}$$

## D. $D = 11$ Gamma-matrices and $D = 10$ Dirac-spinors

We take the $D = 11$ SO$(1,10)$ spin $32 \times 32$ matrices to be in a real Majorana-representation

$$\begin{aligned}
\Gamma^0 &= -i\sigma_2 \otimes I_{16} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} I_8 & 0 \\ 0 & I_8 \end{pmatrix}, \\
\Gamma^a &= \sigma_1 \otimes \gamma^a_{16} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & \gamma^a_8 \\ \gamma^{a,T}_8 & 0 \end{pmatrix}; \quad a = 1,\ldots,8\,, \\
\Gamma^9 &= -\sigma_1 \otimes \gamma^9_{16} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \otimes \begin{pmatrix} I_8 & 0 \\ 0 & -I_8 \end{pmatrix}, \\
\Gamma^{10} &\equiv \Gamma^0\Gamma^1\cdots\Gamma^8\Gamma^9 = \sigma_3 \otimes I_{16} = \\
&= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} I_8 & 0 \\ 0 & I_8 \end{pmatrix} = \begin{pmatrix} I_{16} & 0 \\ 0 & -I_{16} \end{pmatrix},
\end{aligned}$$

where we use $\epsilon_{0\ldots9} = 1 = -\epsilon^{0\cdots9}$. The Dirac-matrices satisfy

$$\left\{\Gamma^M, \Gamma^N\right\} = 2\,\eta^{MN} = 2(-,+,\ldots,+)\,,$$

while the real $16 \times 16$ $\gamma_{16}^a$ submatrices obey the relations

$$\{\gamma_{16}^a, \gamma_{16}^b\} = 2\,\delta^{ab}\,, \qquad \gamma_{16}^{a,T} = \gamma_{16}^a\,, \qquad (\gamma_{16}^a)^2 = I_{16}\,,$$
$$\gamma_{16}^9 \equiv \gamma_{16}^1 \ldots \gamma_{16}^8 = \begin{pmatrix} I_8 & 0 \\ 0 & -I_8 \end{pmatrix}, \qquad \gamma_{16}^{9,T} = \gamma_{16}^9\,, \qquad (\gamma_{16}^9)^2 = I_{16}\,.$$

Finally the $8 \times 8$ submatrices $\gamma^a$ are defined as

$$\begin{aligned}
\gamma^1 &= i\sigma_2 \otimes i\sigma_2 \otimes i\sigma_2\,, & \gamma^2 &= I_2 \otimes \sigma_1 \otimes i\sigma_2\,,\\
\gamma^3 &= I_2 \otimes \sigma_3 \otimes i\sigma_2\,, & \gamma^4 &= \sigma_1 \otimes i\sigma_2 \otimes I_2\,,\\
\gamma^5 &= \sigma_3 \otimes i\sigma_2 \otimes I_2\,, & \gamma^6 &= i\sigma_2 \otimes I_2 \otimes \sigma_1\,,\\
\gamma^7 &= i\sigma_2 \otimes I_2 \otimes \sigma_3\,, & \gamma^8 &= I_2 \otimes I_2 \otimes I_2 = I_8
\end{aligned}$$

and satisfy

$$\gamma^a \gamma^{b,T} + \gamma^b \gamma^{a,T} = 2\delta^{ab}\,, \qquad \gamma^{i,T} = -\gamma^i\,; \quad i = 1, \ldots, 7\,, \qquad \gamma^{8,T} = -\gamma^8\,.$$

$D = 10$ **Weyl-spinor.** We have to deal with ten-dimensional Majorana-Weyl spinors for the gauginos with positive chirality only. For the special ten-dimensional momentum $p_1 = \frac{E}{2}(1, 0, \ldots, 0, 1)$, we find from the Dirac-equation $\Gamma^A \partial_A \lambda(x) = 0$, the following spinor expression in momentum space

$$u_s(p_1) = \sqrt{N} \begin{pmatrix} 0 \\ e_s \\ 0 \\ 0 \end{pmatrix},$$

where $e_s$; $s = 1, \ldots, 8$ denotes the s-th unit vector. In our calculation we actually need the slightly more general spinor corresponding to the ten-dimensional momentum $p = \frac{E}{2}(1, 0, \ldots, 0, \sin\vartheta, \cos\vartheta)$. We can generate $p$ from $p_1$ by a rotation in the 8-9 plane

$$p^A = R_\vartheta p_1^A = \begin{pmatrix} \ddots & 0 & 0 \\ 0 & \cos\vartheta & \sin\vartheta \\ 0 & -\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} p_1^0 \\ \vdots \\ p_1^8 \\ p_1^9 \end{pmatrix}.$$

The corresponding action on the spinor $u_s(p_1)$ is given by

$$\begin{aligned}
u_s(p) = e^{\frac{\vartheta}{2}\Gamma^8\Gamma^9} u_s(p_1) &= \left(\cos\left(\frac{\vartheta}{2}\right) I_{32} + \sin\left(\frac{\vartheta}{2}\right) \Gamma^8\,\Gamma^9\right) u_s(p_1) \\
&= \sqrt{N} \begin{pmatrix} \sin(\vartheta/2)\, e_s \\ \cos(\vartheta/2)\, e_s \\ 0 \\ 0 \end{pmatrix}.
\end{aligned} \tag{D.1}$$

As a convenient normalization choice we choose

$$N \equiv E\,.$$

The charge conjugation matrix $C_{\alpha\beta}$ will be taken as

$$C_{\alpha\beta} = \left(\Gamma^0\right)^{\alpha}{}_{\beta}\,, \qquad C^{\alpha\beta} = \left(\Gamma^{0,-1}\right)^{\alpha}{}_{\beta} = \left(\Gamma^{0,T}\right)^{\alpha}{}_{\beta} = -\left(\Gamma^0\right)^{\alpha}{}_{\beta}\,.$$

**Symmetry properties of Bilinears.** For arbitrary momenta $p$ and $p'$ one obtains

$$\bar{u}_s(p)\Gamma^A u_{s'}(p') = \bar{u}_{s'}(p')\Gamma^A u_s(p)\,, \tag{D.2}$$

$$\bar{u}_s(p)\Gamma^{ABC} u_{s'}(p') = -\bar{u}_{s'}(p')\Gamma^{ABC} u_s(p)\,, \tag{D.3}$$

$$\bar{u}_s(p)\Gamma^{ABCDE} u_{s'}(p') = \bar{u}_{s'}(p')\Gamma^{ABCDE} u_s(p)\,. \tag{D.4}$$

## E. Hyperplane gauge field operators

**Fourier-decomposition of the field operators.**

$$B_A^a(x) = \int \frac{d^9k}{(2\pi)^9\, 2k^0} \sum_{\lambda=1,\ldots,8} \epsilon_A(k,\lambda) \left[b_\lambda^a\,(k)\, e^{ikx} + b_\lambda^{a,\dagger}\,(k)\, e^{-ikx}\right], \tag{E.1}$$

$$\lambda^a(x) = \int \frac{d^9k}{(2\pi)^9\, 2k^0} \sum_{s=1,\ldots,8} u_s(k) \left[d_s^a(k) e^{ikx} + d_s^{a,\dagger}(k) e^{-ikx}\right]. \tag{E.2}$$

**Anti-/commutators:**

$$\left[b_\lambda^a(k), b_{\lambda'}^{b,\dagger}(k')\right] = \delta^9(k-k')\delta_{\lambda\lambda'}\delta^{ab}(2\pi)^9 2k^0\,, \tag{E.3}$$

$$\left[b_\lambda^a(k), b_{\lambda'}^{b}(k')\right] = \left[b_\lambda^{a,\dagger}(k), b_{\lambda'}^{b,\dagger}(k')\right] = 0\,, \tag{E.4}$$

$$\left\{d_s^a(k), d_{s'}^{b,\dagger}(k')\right\} = \delta^9(k-k')\delta_{ss'}\delta^{ab}(2\pi)^9 2k^0\,, \tag{E.5}$$

$$\left\{d_s^a(k), d_{s'}^{b}(k')\right\} = \left\{d_s^{a,\dagger}(k), b_{s'}^{b,\dagger}(k')\right\} = 0\,. \tag{E.6}$$

**Wick-contractions:**

$$\underbracket{b_\lambda^a(p)\, B_A^b}\,(x) = \delta^{ab}\epsilon_A(p,\lambda)\, e^{-ipx}\,, \tag{E.7}$$

$$\underbracket{B_A^a(x)\, b_\lambda^{b,\dagger}}\,(p) = \delta^{ab}\epsilon_A(p,\lambda)\, e^{ipx}\,, \tag{E.8}$$

$$\underbracket{d_s^a(p)\, \lambda^{b\alpha}}(x) = \delta^{ab}u_s^\alpha\,(p)\, e^{-ipx}\,, \tag{E.9}$$

$$\underbracket{\lambda^{a\alpha}(x) d_s^{b,\dagger}}(p) = \delta^{ab}u_s^\alpha\,(p)\, e^{ipx}\,. \tag{E.10}$$

## References


[1] P. Hořava and E. Witten, *Heterotic and type-I string dynamics from eleven dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209].

[2] P. Hořava and E. Witten, *Eleven-dimensional supergravity on a manifold with boundary*, *Nucl. Phys.* **B 475** (1996) 94 [hep-th/9603142].

[3] E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, *Nucl. Phys.* **B 471** (1996) 135 [hep-th/9602070].

[4] H.P. Nilles, M. Olechowski and M. Yamaguchi, *Supersymmetry breaking and soft terms in M-theory*, *Phys. Lett.* **B 415** (1997) 24 [hep-th/9707143].

[5] J. Polchinski, *Tasi lectures on D-branes*, hep-th/9611050;
C.P. Bachas, *Lectures on D-branes*, hep-th/9806199.

[6] E. Cremmer, B. Julia and J. Scherk, *Supergravity theory in 11 dimensions*, *Phys. Lett.* **B 76** (1978) 409.

[7] J.O. Conrad, *Brane tensions and coupling constants from within M-theory*, *Phys. Lett.* **B 421** (1998) 119 [hep-th/9708031].

[8] B.S. DeWitt, *Quantum theory of gravity, 2. The manifestly covariant theory*, *Phys. Rev.* **D 162** (1967) 1195.

[9] A. Das and D.Z. Freedman, *Gauge quantization for spin* 3/2 *fields*, *Nucl. Phys.* **B 114** (1976) 271.

[10] M.R. Garousi and R.C. Myers, *Superstring scattering from D-branes*, *Nucl. Phys.* **B 475** (1996) 193 [hep-th/9603194].

[11] A. Hashimoto and I.R. Klebanov, *Decay of excited D-branes*, *Phys. Lett.* **B 381** (1996) 437 [hep-th/9604065].

[12] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory*, Cambridge University Press, Cambridge 1987.